# COFFEE IMAGES: A STUDY IN THE SIMULTANEOUS DISPLAY OF MULTIVARIATE QUANTITATIVE AND CATEGORICAL VARIABLES FOR SEVERAL ASSESSORS

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SUMMARY

The paper discusses a small image study in which seven assessors judge nine brands of coffee in terms of six quantitative variables and five categorical variables. Generalised Procrustes Analysis and Generalised Biplots are combined to display simultaneously information on the brands and on both quantitative and categorical variables. An outline is given of the methodology.

*Keywords*: Categorical variables; Consumer research; Generalised biplots; Generalised Procrustes Analysis; Image studies; Metric scaling; Multivariate analysis

## 1. Introduction

In consumer research, a panel of assessors is often asked to give judgements on the characteristics of some product under consideration. Judgements may be of diverse kinds and the scales used may include both quantitative and qualitative measurements. Quantitative scales are rarely measured directly but the members of the panel will be asked to use, say, a ten-point scale or to indicate the strength of their response to some question by marking an appropriate distance along a line. Qualitative responses are selected from a list of attributes (e.g. red, green, yellow, blue for colour). Often attributes are ordered (e.g. low, middle and high income) but in the following we take no cognizance of this information. Occasionally each individual member of the panel chooses his or her own attributes (e.g. in Free Choice Profiling, FCP, Williams and Langron 1984). Rather than write about attributes and characteristics we use the standard statistical methodology, referring to qualitative variables as categorical variables and to their attributes as category-levels. With FCP not only do the variables differ between individuals but also the number of variables, thus making it difficult to compare the responses of different individuals.

Generalised Procrustes Analysis (GPA, Gower, 1975) offers a way of comparing individuals who judge products using different variables. Its basic assumption is that a distance can be defined so that distances between two products, as judged by two individuals, are comparable, even though they may be based on different variables. When the variables are nominally the same, there is no guarantee that two individuals perceive them in the same way, so supporting the use of GPA in a wider context than just with FCP-like data (e.g. Dijksterhuis and Punter, 1990). This is the justification for using GPA in the following, where the variables used by all assessors are nominally the same. The distance assumption allows a map to be made of the products for each individual. Sets of maps may be matched, in an obvious least-squares sense, and compared as is described in Section 3.

In the use of GPA described here, the distance between pairs of products is defined (see Section 3) as a function of the values taken by the quantitative and/or categorical variables used. The original description of GPA did not discuss how information on the original variables could be included in the maps, but soon (Arnold and Williams, 1985) quantitative variables were expressed as vectors through a common origin in what amounted to an application of classical biplot methodology (Gabriel, 1971). Methods such as multiple correspondence analysis (MCA) show information on categorical variables but only when all the variables have qualitative form and only for one individual. Recent advances in biplot theory (Gower and Harding, 1988; Gower, 1991,1992) unify the treatment of quantitative and categorical variables and offer other generalisations; for example, MCA is a special case. The methodology used in this paper brings together GPA and a useful special case of Generalised Biplots (GBP, Gower, 1992).

## 2. Data

Seven assessors were asked to judge nine brands of coffee on five categorical and six quantitative variables. The assessors, two men and five women, were presented with a package of each coffee and were asked several questions. The coffees were not tasted, but the assessors were asked to respond according to their conceptions of the properties of the coffee. Thus their responses would be based on their pre-knowledge of the coffees as elicited by the packaging itself. Table 1 briefly describes the nine coffees together with their prices; Table 2 presents the questions.

Table 1
The nine kinds of coffee used in the experiment with their price in Dutch Guilders per 250 gram. The symbols represent the corresponding coffees in the figures.

| Symbol | Coffee | Description | Price |
|---|---|---|---|
| △ | Red brand | Ordinary coffee | 2.54 |
| ★ | Golden brand | Luxury coffee | 3.64 |
| + | Moccona | Instant coffee | 12.59 |
| × | Nescafe | Instant coffee | 14.25 |
| □ | B-Brand | Cheap coffee | 1.25 |
| ○ | Hag | Decafinated coffee | 3.13 |
| ◇ | Max Havelaar | Third world coffee | 3.58 |
| ■ | Espresso | Espresso coffee | 3.69 |
| ● | Chocolate flavour | Coffee with chocolate flavour | 11.75 |

To answer question 6, the assessors were asked to give the price in Dutch Cents (100 Cents = One Guilder) they would be willing to pay for 250g of the coffee. Questions 7 to 11 were scored on line-scales, giving scores ranging from 0 to 100. Each subject was asked the same questions but, for the reason given in Section 1, it was thought appropriate to treat the data as if they were of FCP form .

Data of these kinds are collected in the hope of answering questions about the homogeneity, or otherwise, of the patterns of response across individuals. With homogeneous patterns, supplementary problems are to suggest which variables are important and which unimportant in determining the responses, and to examine the extent of departures from an average response. With heterogeneous patterns, it is of interest to ask if there is evidence that the individuals fall into two or more homogeneous groups. Because we had so few assessors, there was insufficient information to answer most questions of these kinds and this exposition must be regarded as a pilot-study to validate the methods proposed.

Table 2
The questions asked of the assessors. The underlined letters and words can be found in the figures together with the indicated abbreviations of the category-level names.

| | QUESTIONS ON CATEGORICAL VARIABLES | CATEGORY-LEVELS |
|---|---|---|
| 1 | How often do you Drink this coffee? | |
| | | Dn never |
| | | Ds sometimes |
| | | Dr regularly |
| | | Do often |
| | | Da always |
| 2 | What is the most suitable Moment for this coffee? | |
| | | Mb with breakfast |
| | | Mm the morning |
| | | Ml with lunch |
| | | Ma the afternoon |
| | | Md after dinner |
| | | Me the evening |
| 3 | What is the most suitable Occasion for this coffee? | |
| | | Oh at home (each day) |
| | | Ow at work |
| | | Ov during vacations |
| | | Or in a restaurant/café |
| | | Op at week-ends/public holidays |
| | | Od after dinner |
| 4 | Which Income-group buys this coffee? | |
| | | Il low incomes |
| | | Im middle incomes |
| | | Ih high incomes |
| 5 | Would you Buy this coffee? | |
| | | Bn never |
| | | Bs sometimes |
| | | Br regularly |
| | | Bo often |
| | | Ba always |

| | QUESTIONS ON QUANTITATIVE VARIABLES | | | |
|---|---|---|---|---|
| 6 | What price are you willing to pay for 250g of this coffee ? | | | |
| 7 | Amount of odour? | weak | - | strong |
| 8 | Amount of taste or aroma? | weak | - | strong |
| 9 | Full-flavouredness/raciness? | weak | - | strong |
| 10 | Bitterness? | weak | - | strong |
| 11 | Quality? | bad | - | good |

## 3. Methodology

We may imagine data-matrices $\mathbf{X}_k$ ($k = 1,2,\dots,K$) to be available for $K$ individuals. A typical element of $\mathbf{X}_k$ will be written $x_{ijk}$, where $i = 1,2,\dots,N$ refers to $N$ objects (coffee brands in the example discussed below) which are the same for all $K$ individuals and $j = 1,2,\dots,J_k$ refers to variables. The matrix $\mathbf{X}_k$ has $J_k$ columns which refer to the variables chosen by the $k$th individual; in general these will differ from individual to individual but in our example, $J_k$ is constant. The variables may be quantitative, categorical, or a mixture of both types.

Distances $d_{rsk}$ between all pairs $r,s$ of objects are defined (see §3.3) between rows $r$ and $s$ of $\mathbf{X}_k$, where $r,s = 1,2,\dots,N$. These may be collected into a symmetric matrix $\mathbf{D}_k = \{d_{rsk}\}$ with zero diagonal. From $\mathbf{D}_k$ a configuration with coordinates given by the rows of a matrix $\mathbf{Y}_k$ may be obtained by any desired form of metric or non-metric scaling. In our example we use principal coordinate analysis PCO/classical scaling which, with the distances used here, always gives real Cartesian coordinates $\mathbf{Y}_k$ in, at most, $N - 1$ dimensions, which generate the distances $d_{rsk}$ exactly.

### *3.1. Overview of Generalised Procrustes Analysis*

The multidimensional-scaling configurations $\mathbf{Y}_k$ ($k = 1,2,\dots,K$) are used as the basis of GPA. That is, orthogonal matrices $\mathbf{H}_k$ and, possibly, scaling factors $\rho_k$ are found which minimise $\sum_{h<k}^{K} \|\rho_k \mathbf{Y}_k \mathbf{H}_k - \rho_h \mathbf{Y}_h \mathbf{H}_h\|_2$, or, what is the same thing, minimise $\sum_{k=1}^{K} \|\rho_k \mathbf{Y}_k \mathbf{H}_k - \mathbf{Y}\|_2$, where $\|\mathbf{A}\|_2 = \text{Trace}(\mathbf{A}'\mathbf{A})$ and $\mathbf{Y}$ is the GPA group-average given by $\mathbf{Y} = \frac{1}{K}\sum_{k=1}^{K}(\rho_k \mathbf{Y}_k \mathbf{H}_k)$. Thus, the orthogonal matrices $\mathbf{H}_k$ represent generalised rotations which are chosen to optimise the overall match of the individual configurations to their average. The scaling factors $\rho_k$ allow for the possibility that there are size differences between the configurations; a suitable constraint is chosen to exclude trivial solutions with all $\rho_k = 0$. All this is familiar, but we need to establish the notation and draw attention to the formulation that distinguishes the raw data $\mathbf{X}_k$ from the configurations $\mathbf{Y}_k$. In many applications the two may be taken to be the same, but we wish to emphasise that this is not a necessary constraint; the formulation adopted here allows distances $d_{rsk}$ to be defined very generally and it is this that allows for the possibility of including categorical variables, as will be described below. When $\mathbf{Y}_k =$

$\mathbf{X}_k$, there is an implicit assumption that Pythagorean distance given by $d^2_{rsk} = \sum_{j=1}^{J_k}(x_{rjk} - x_{sjk})^2$ is used. However distance is defined, some preliminary normalisation of the variables of $\mathbf{X}_k$ may be necessary to ensure commensurability; it is assumed that all necessary pre-scaling of this kind has been done (see e.g. Dijksterhuis and Gower, 1992). When pre-scaling is applied, there is little justification for including the additional scaling factors, $\rho_k$.

### *3.2. Components Analysis, Linear Biplots and GPA*

In a principal components analysis of the data for the *k*th individual, the biplot methodology for exhibiting variables in the graphical display involves the construction of the component loadings $\mathbf{V}_k$ that satisfy the eigenvalue equation

$$(\mathbf{X}_k'\mathbf{X}_k)\mathbf{V}_k = \mathbf{V}_k\mathbf{\Lambda}_k,$$

where $\mathbf{X}_k$ is now assumed to be expressed in deviations from the sample-means. Thus $\mathbf{V}_k$ is orthogonal ($\mathbf{V}_k'\mathbf{V}_k = \mathbf{I}$, a unit matrix) and $\mathbf{\Lambda}_k$ is diagonal (the matrix of eigenvalues). The *i*th object has coordinates given by the *i*th row of $\mathbf{Y}_k = \mathbf{X}_k\mathbf{V}_k$. The vector plotted for the *j*th variable is obtained by plotting the point whose coordinates are given by the *j*th row of $\mathbf{V}_k$, and joining this to the origin. This vector is termed the *j*th biplot axis. Both the component analysis and the biplot axes are usually plotted in some small number *r* of dimensions, often $r = 2$, that is obtained by using $\mathbf{V}_{k_r}$, the first *r* columns of $\mathbf{V}_k$. Just as the original axes may have associated scales of measurement, so may the biplot axes. On the biplot axes, one unit of measurement may be taken to be (i) that given by $\mathbf{V}_k$ as described above, which we refer to as the *interpolation scale* and (ii) that given by $\text{diag}(\mathbf{V}_{k_r}\mathbf{V}_{k_r}')\mathbf{V}_{k_r}$, which we refer to as the *prediction scale.* The scales given by (i) and (ii) define just one unit of measurement, and just as when plotting with conventional axes, a series of *markers* may be associated with the biplot axes indicating one unit, two units, three units and so on. To *interpolate* a new sample or product, take the vector-sum of the markers on the interpolation scales representing the values required for the variables. To *predict* the values associated with a sample-point in the ordination, drop perpendiculars from the point and read off the values against the prediction scale markers. In exact representations the two scales are arranged so that interpolating a set of predicted values recovers the original sample-point and the predicted values to be associated with an interpolated point are the same as the values used for the interpolation; the two scales are consistent. This consistentcy property is lost in approximations (see Gower, 1991 for a more detailed explanation).

Now, suppose biplot axes have been computed as described, then we can regard the coordinate points on the biplot axes given by the rows of $\mathbf{V}_k$ as being rigidly embedded in the components space, together with the object-points $\mathbf{Y}_k = \rho_k \mathbf{X}_k \mathbf{V}_k$. When GPA rotates $\mathbf{Y}_k$ through $\mathbf{H}_k$, then $\mathbf{V}_k$ is rotated to $\mathbf{V}_k\mathbf{H}_k$ giving biplot axes embedded in the configurations generated by the GPA analysis. Corresponding to the group-average $\mathbf{Y}$ of objects, is the group average $\mathbf{Z} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{V}_k\mathbf{H}_k$ which gives the biplot axes to be associated with the group-average configuration. This, however, is valid only when the same variables pertain to all the individuals. With FCP, combining incommensurable variables in this way is not valid. Indeed, the dimensions of the matrices $\mathbf{V}_k$ will generally differ from individual to individual. Even when averaging variables is not permissible, two or more variables may be seen to have similar directions, and then one might provisionally regard this as an indication that these variables might pertain to the same, or similar, underlying factors.

### *3.3. Generalised Biplots*

The above sketches what is reasonably well-known. The question arises whether or not something similar can be done for forms of multidimensional scaling other than components analysis, and for distances other than Pythagorean. The basic methodology for this extension is given by Gower and Harding (1988) who described non-linear biplots (NLB), that can be used for any form of distance defined on quantitative variables, and by Gower (1992) who described Generalised Biplots (GBP), which further extends the methodology to include categorical variables. GBP can be very general indeed but here we confine ourselves to a special case, which itself has a considerable degree of generality. The results required are stated below; derivations and proofs are given in Gower (1992).

We assume that each variable contributes independently to overall squared-distance. That is:

$$d^2_{rsk} = \sum_{j=1}^{J_k} g_j(x_{rjk}, x_{sjk}) \tag{1}$$

where $g_j(.,.)$ is a function that determines the distance between two samples for the $j$th variable alone. This general form will be required later but for the present it is assumed that $g_j(.,.)$ is the same function for all variables and hence may be written $g(.,.)$. The form (1) includes the chi-squared distance of MCA, Pythagorean distance and many popular dissimilarity coefficients.

Next, suppose the rows of $\mathbf{Y}_k$ contain the Euclidean coordinates for the $k$th individual, generated by some form of ordination as described at the begining of §3. Then the coordinates of the marker $\xi$ on the $j$th biplot axis relative to the axes of the ordination $\mathbf{Y}$ is given by:

$$\mathbf{y}(\xi) = -\frac{1}{2}(\mathbf{YY}')^{-1}\mathbf{Y}'(\mathbf{f} + 2\mathbf{D}_k\mathbf{1}/n) \tag{2}$$

where $\mathbf{f} = \{x_{ij}, \xi\}$, is the vector giving the distance of the marker from each of the $N$ objects ($i = 1,2,...,N$).

As $\xi$ varies $\mathbf{y}(\xi)$ will trace out a non-linear trajectory which corresponds to the $j$th biplot axis. When $g(x_{rjk}, x_{sjk}) = (x_{rjk} - x_{sjk})^2$ for all $j$, we have Pythagorean distance and if, additionally, PCO is the ordination method that is used, then we have components analysis and classical linear biplots as a special case. In the examples, we assume this special form, so our biplots for quantitative variables are identical with those of the well-known classical linear biplots. However, GBP also allows categorical variables. Distances for categorical variables may be defined in many ways. Here we assume the extended matching coefficient:

$$g(x_{rjk}, x_{sjk}) \quad \left.\begin{array}{l} = 0 \text{ when } x_{rjk} = x_{sjk} \\ = 1 \text{ otherwise} \end{array}\right\}. \tag{3}$$

Thus, if the $j$th variable is, say, a three-level categorical variable, of the colours red, green and blue then two objects contribute zero distance if they are the same colour, and unit distance if they are different colours. With this simple definition, Gower (1992) shows that the coordinates that represent the $j$th categorical variable are given by:

$$\mathbf{Z}_j' = \mathbf{\Lambda}^{-1}\mathbf{Y}'\mathbf{G}_j(\tfrac{1}{N}\mathbf{C}_j\mathbf{1}\,\mathbf{1}' - \mathbf{I}) \tag{4}$$

where $\mathbf{G}_j$ is the indicator matrix for the $j$th variable (i.e. $\mathbf{G}_j(i,l) = 1$ when level $l$ occurs for the $i$th object, else is zero) and $\mathbf{C}_j = \text{diag}(\mathbf{G}_j'\mathbf{G}_j)$ gives the number of occurrences of the different levels of the $j$th variable. Here, and in the formulae that follow, there are $J_k$ of these matrices for the kth individual. Thus, when $J_k = J$, a constant, there are $JK$ sets of coordinates when totalled over all individuals.

One may note the similarity between (4) and the formula for category coordinates in MCA:

$$\mathbf{Z}_j' = \mathbf{\Sigma}^{-1}\mathbf{Y}'\mathbf{G}_j\mathbf{C}_j^{-1} \tag{5}$$

for which

$$d_{rsk}^2 = \frac{1}{J_k^2}\sum_{j=1}^{J_k}\left(\frac{1}{c_{jr}} + \frac{1}{c_{js}}\right), \tag{6}$$

where $c_{jr}(c_{js})$ gives the number of occurrences of the category-level of the $j$th variable observed for the $r$th($s$th) sample for the $k$th individual and $\mathbf{\Sigma}^2 = \mathbf{\Lambda}$.

A third possbility is to use the GBP methodology with (1) defined by the chi-squared distance (6). This gives:

$$\mathbf{Z}_j' = \frac{1}{J_k^2}\boldsymbol{\Lambda}^{-1}\mathbf{Y}'\mathbf{G}_j\mathbf{C}_j^{-1} \quad (7)$$

which differs from (5) only in replacing the scaling factors $\boldsymbol{\Sigma}^{-1}$ by $\frac{1}{J_k^2}\boldsymbol{\Lambda}^{-1}$. However, we agree with the criticism of Greenacre (1989) that chi-squared distance with the implicit upweighting of scarce categories relative to frequent categories given by (6), is not an attractive distance to use with sparse categorical data. For its simplicity, and for other reasons given by Gower (1992), we prefer the extended matching coefficient and hence used the category coordinates given by (4) in our example. However, the methods based on (4), (5) or (6) have much in common and are included within the general framework. In all cases, the weighted mean of the category-level points of any variable is at the origin/centroid, i.e. $\mathbf{1}'\mathbf{C}_j\mathbf{Z}_j = 0$.

A major advantage of GBP is the way that (2) allows quantitative and categorical variables to be included in the same analysis. All that is required is that in the distance given by (1), $g_j(x_{rjk}, x_{sjk})$ be defined as Pythagorean for quantitative variables and as the extended matching coefficient for categorical variables. These are the definitions we happen to have used; any other combination of distance definitions that might be deemed desirable may be substituted and in extreme cases every term of (1) could be defined differently.

Thus, to combine both categorical and quantitative variables in one analysis, a PCO was carried out on each $\mathbf{X}_k$, defining squared distances by (1) with the first five terms based on the extended matching coefficient and the remaining six on Pythagorean distances. Such a combined analysis, reduces to an eigenvalue decomposition of each of the $K$ matrices

$$1/2\mathbf{B}_k + \mathbf{Q}_k\mathbf{Q}_k' = \mathbf{Y}_k\mathbf{Y}_k', \text{ with } \mathbf{Y}_k'\mathbf{Y}_k \text{ diagonal} \quad (8)$$

where

$$\mathbf{B}_k = \frac{1}{2}(\mathbf{I} - \mathbf{N})\mathbf{G}^{(k)}\mathbf{G}^{(k)\prime}(\mathbf{I} - \mathbf{N}) \quad (9)$$

with $\mathbf{Q}_k$ being the mean-centred submatrix of $\mathbf{X}_k$ containing the quantitative variables and $\mathbf{N} = \mathbf{I} - \mathbf{1}\mathbf{1}'/N$, where $\mathbf{1}$ is a vector of $N$ units and $\mathbf{G}^{(k)}$ is the indicator matrix for all the categorical variables for the $k$th assessor. That is $\mathbf{G}^{(k)} = (\mathbf{G}_1^{(k)}, \mathbf{G}_2^{(k)}, \ldots, \mathbf{G}_5^{(k)})$, where $\mathbf{G}_j^{(k)}$ is the indicator matrix for the $j$th categorical variable as used in (4).

Thus with $\mathbf{Y}_k$ defined by (8), (4) gives the coordinates of the points to plot for the category levels. The corresponding formula for quantitative variables is:

$$\mathbf{Z}_k' = \Lambda^{-1}\mathbf{Y}_k'\mathbf{x}_{jk}\mathbf{x}_{jk}' \qquad (10)$$

This will give coordinates for the observed values of the variable $\mathbf{x}_{jk}$ (i.e. the *j*th column of $\mathbf{X}_k$, the data-matrix for the *k*th individual) and these will be collinear because $\mathbf{Z}_k$, given by (10), has unit rank because it is the product of the two vectors $\Lambda^{-1}\mathbf{Y}_k'\mathbf{x}_{jk}$ and $\mathbf{x}_{jk}'$. Only one point is needed to define each linear biplot axis but (10) gives *N* collinear points. A single point, representing the marker for one unit of the *j*th variable, has coordinates $\Lambda^{-1}\mathbf{Y}_k'\mathbf{x}_{jk}$; the other markers are equally spaced along the biplot axis. With non-Pythagorean, but Euclidean, distances, the trajectory is non-linear and even the *N* points corresponding to the data-values of a variable given by the general form of (4) and (10) may be insufficient to give good resolution. Then one would have to use (2) to interpolate as many points as were needed for adequate definition (Gower, 1991).

## 4. Analyses

The analyses are presented as follows. First we give GPA analyses for the seven assessors, based on PCO analyses using (i) only the categorical variables and (ii) all the variables; in both cases, information is included on the variables. We conclude with a novel form of GPA which is permissible only when individual assessors use the same category-levels.

### 4.1. *Categorical Variables*

Table 3(a) gives the percentage variance explained by the first two dimensions of the PCO solutions using only the categorical variables and defining distance by the extended matching coefficient (3). The scores of assessor 3 capture most variance (63.4%) and those of assessor 5 capture least (54.1%) in the first two dimensions. These percentages might appear disappointing but are typical for work in this field. Figure 1 shows the two-dimensional PCOs for these two assessors. Comparison is difficult because of the arbitrary relative orientations of the two configurations. However, there are clear differences between the relative positions both of the brands and of the category-levels. Two general comments are prompted by Figure 1. Firstly, it can be seen that neither assessor uses all 25 category-levels to describe the coffee; this is typical but it can make for difficulties in comparing configurations. Secondly, the three category levels for income, which one would expect to be a clear case of an ordered categorical variables, are shown as at the vertices of a near equilateral triangle by assessor 3 instead of as an approximate linear ordering. This reflects the multidimensional nature of the extended matching coefficient, which allocates equal distance to the difference between low and high income as to the difference between middle and high income. Table 3(a) and also the PCO plots showed no association with gender, so this aspect is not explored further in the following.

Table 3
Percentage explained variance for the first two dimensions of the PCO analyses of the categorical variables of the seven assessors. Female f, Male m.

| (a) | Categorical Variables | | | | (b) | All Variables | | |
|---|---|---|---|---|---|---|---|---|
| | Assessor | dim 1 | dim 2 | Sum | Assessor | dim 1 | dim 2 | Sum |
| f | 1 | 36.2 | 21.0 | 57.2 | 1 | 37.2 | 25.3 | 62.5 |
| f | 2 | 39.0 | 23.4 | 62.4 | 2 | 34.3 | 22.9 | 57.2 |
| m | 3 | 40.6 | 22.8 | 63.4 | 3 | 49.0 | 21.9 | 70.9 |
| f | 4 | 34.3 | 23.5 | 57.8 | 4 | 46.4 | 19.4 | 65.8 |
| f | 5 | 30.2 | 23.9 | 54.1 | 5 | 35.7 | 18.4 | 54.1 |
| m | 6 | 31.0 | 23.7 | 54.7 | 6 | 43.4 | 19.6 | 63.0 |
| f | 7 | 34.8 | 22.1 | 56.9 | 7 | 42.5 | 19.8 | 62.3 |

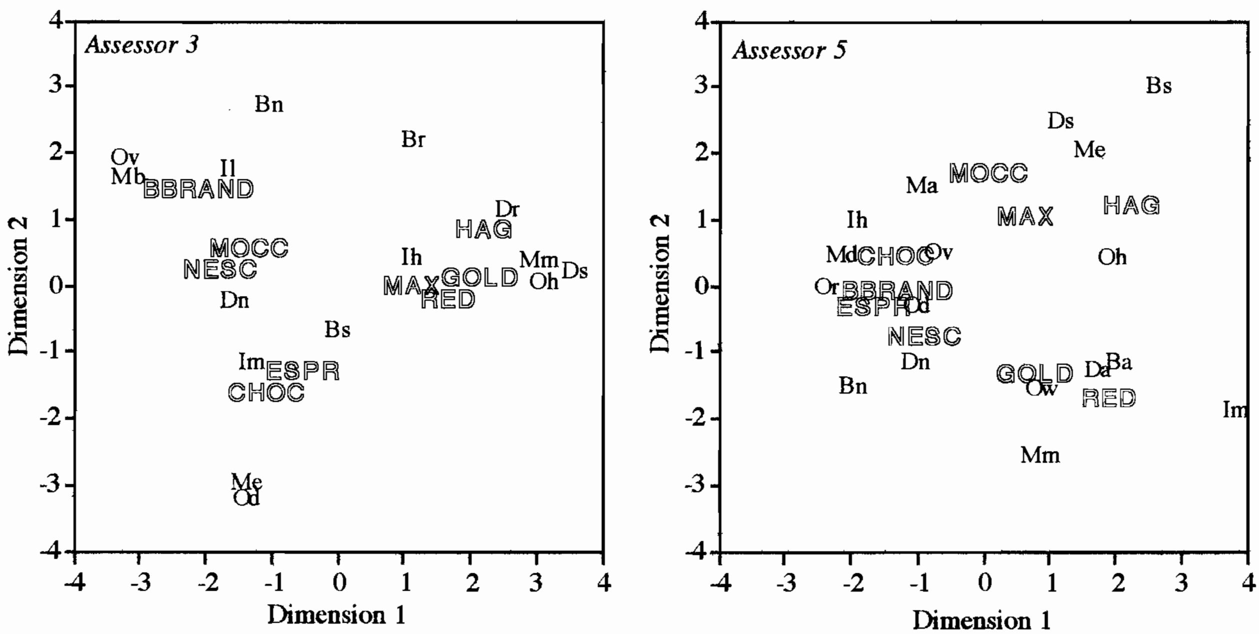



Figure 1
The two-dimensional PCO approximations for assessors 3 and 5 using only categorical variables. The category-points contain the first capital letter of the question from which they stem and a short form of the category-level (see Table 2).

The role of GPA in minimising the effects of arbitrary orientations is discussed in the following. First the seven configurations for the brands obtained by PCO are oriented to best fit as described in Section 3; simultaneously the category-level points were given the same rotations. This could have been done on the two-dimensional approximations but we have used all eight dimensions required to give an exact representation of the nine brands, so as not to sacrifice information at this stage. To ensure commensurability, the configurations were first scaled to equal sum-of-squares; no isotropic scaling factors were fitted. Figure 2 gives a two-dimensional display for each assessor, where orientation is relative to the principal axes of the GPA group-average and not to its own principal axes; this makes it easier to compare the configurations of the assessors and accounts for the fact that the two-dimensional configurations of assessors 3 and 5 are not rotations of the corresponding configurations of Figure 1. Figure 2 also shows the group-average, which is merely the average of the configurations for the seven assessors. The configurations for assessors 3 and 5 may be compared with those given in Figure 1. As expected, the approximations differ but, apart from orientation, the configurations for assessor 3 have much in common; those for assessor 5 do not agree well but this is not unexpected as this assessor seems to lie in a different space to the others.

Table 4
Percentage sums-of-squares of the GPA analyses shown in Figures 2, 3 and 4. The variation in the two-dimensions exhibited in the figures is shown separately from the remaining six dimensions. The quantity minimised by GPA is the total residual sum-of-squares.

ANALYSES OF VARIANCE

(a) *Categorical Variables Only*

| | Exhibited 2-Dimensions | Non-Exhibited 6-Dimensions | Total |
|---|---|---|---|
| Group Average | 41.47 | 27.43 | 68.91 |
| Residual | 14.06 | 17.03 | 31.09 |
| Total | 55.54 | 44.46 | 100 |

(b) *Categorical and Quantitative Variables*

| | Exhibited 2-Dimensions | Non-Exhibited 6-Dimensions | Total |
|---|---|---|---|
| Group Average | 49.29 | 19.60 | 68.89 |
| Residual | 19.74 | 11.36 | 31.11 |
| Total | 69.04 | 30.96 | 100 |

(c) *Combined Analysis of Assessors and Category-Levels*

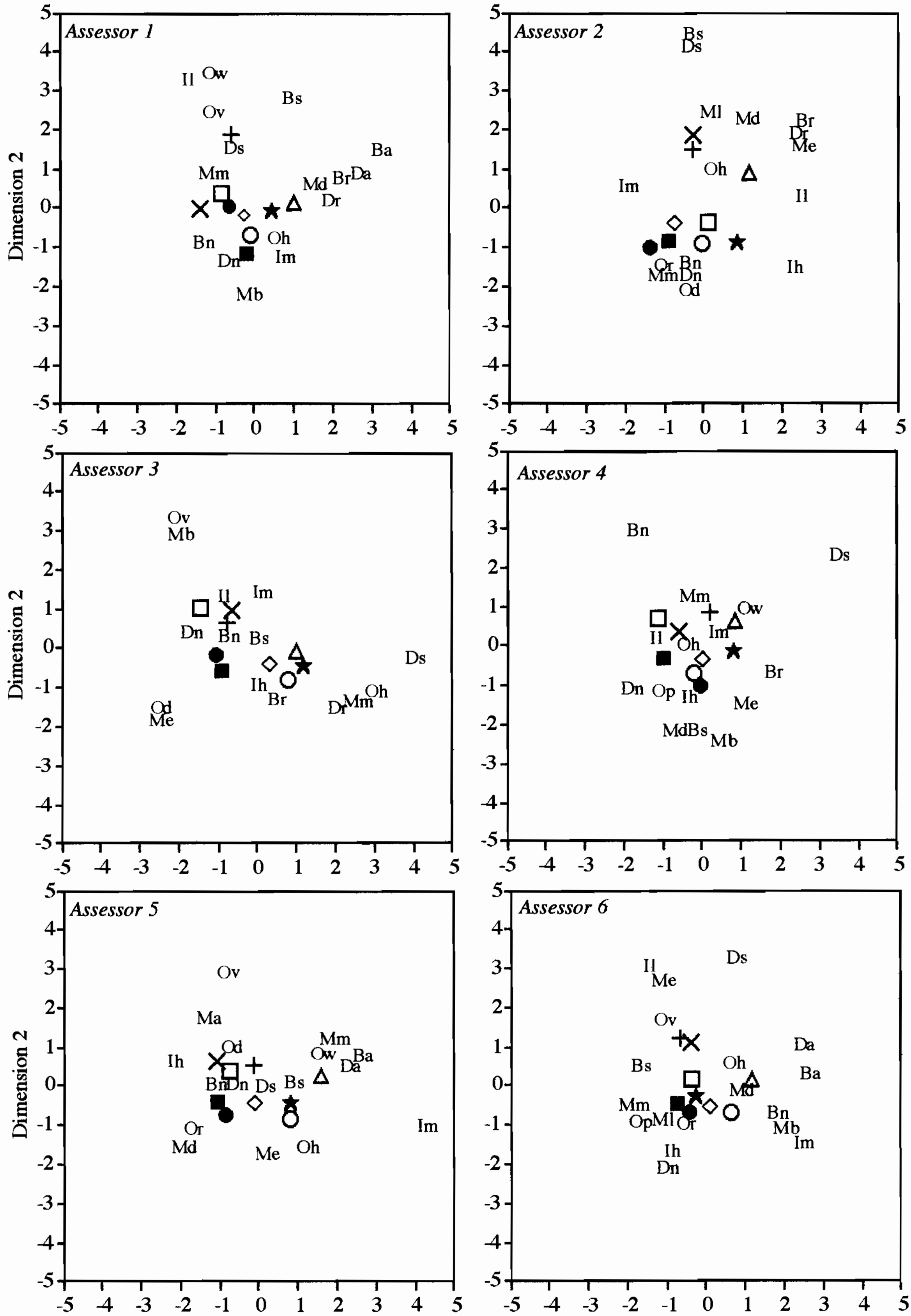
| | Exhibited 2-Dimensions | Non-Exhibited 6-Dimensions | Total |
|---|---|---|---|
| Group Average | 30.34 | 35.51 | 65.85 |

| | | | |
|---|---|---|---|
| Residual | 13.23 | 20.92 | 34.15 |
| Total | 43.57 | 56.43 | 100 |

The whole of a GPA can be summarised in an analysis of variance and this is done in Table 4a where, for convenience, the contributions are expressed as a percentage of the total sum of squares. Everything is in eight dimensions but it is desirable to separate the variation in the two dimensions exhibited in the figures from the remaining six, and this has been done in Table 4. It can be seen from the total row, that the two dimensions exhibited account for rather more than the six dimensions not exhibited and, although dimensionality cannot be equated to degrees of freedom in the normal way, this indicates that Figure 2 is exhibiting about four times as much variation per dimension than is occurring in the unexhibited space. The residual sum-of-squares represents the divergence of the individual assessors from the group-average. The quantity minimised by GPA is the residual sum-of-squares in the total space, which in this case is 31.09 percent of the total variation, rather less than half of which is in the exhibited space. The Group average, which may be thought of as the "signal", is much better represented in the exhibited space than in the unexhibited space; on a per-dimension basis over four and a half times better. The variation of the group-average could be broken down into the individual contributions of each assessor but, mainly for lack of space, this is not shown. A full account of analysis of variance in the context of GPA and allied methods is given by Dijksterhuis and Gower (1991).

Turning again to Figure 2, the same groups of coffees can be identified, although they show much more clearly in Figure 1 than in Figure 2 where they have suffered as a result of the GPA fitting. The configurations in Figure 2 are rotated to best fit the group-average and when the assessors have heterogeneous responses, the group-average is likely not to show clear differentiation of all the objects by assigning all the objects to positions close to the centre. The detailed structure of the configuration for an assessor is then liable to be diluted in the attempt to match an average that is weak in this assessor's structure. There seems to be a tendency of this kind in the coffee data but sufficient structure remains to support some tentative remarks. Moccona goes with Nescafé, in almost all plots; they are both instant coffee's. The cheap B-Brand coffee tends to lie apart from all other coffees, probably reflecting its poor image; it is judged like the instant coffee's Mocc and Nesc. In Holland instant coffee has a poor image. It is used on vacations (Ov) or at breakfast (Mb) by assessor 3. Espresso seems to share properties with Chocolate coffee for assessor 3, it is drunk after dinner (Od), in the evening (Me). The other coffees (Red, Max, Gold, Hag) are sometimes or regularly drunk (Ds, Dr), at home (Oh), in the morning (Mm) by assessor 3. For the same assessor the categories buy never and buy regularly (Bn, Br) appear at different positions in Figures 1 and 2; apparently these differences are submerged in the comparison with the other assessors by GPA.

When each configuration is compared with the group-average, it can be seen that the assessors do not form a homogeneous group. However, there is some evidence that some category-points apply to some brands rather consistently for all assessors. This can be seen in the Group Average plot, in which the category-points that are used most consistently lie at the outer part of the plot. There seems to be little difference in the occasions 'after dinner', 'at week-ends/public holidays' and 'in a restaurant/café', the category-points Or, Od, Op lie close together. These occasions apply mainly to the coffees in that part of the figure, i.e. Hag, Choc, and Espr. These coffees are judged to be mainly bought by those with high incomes (Ih) and are never drunk by the assessors themselves (Dn). The instant coffees Mocc and Nesc lie together with BBrand and are used on vacations (Ov), in the afternoon or with lunch (Ma, Ml), and bought by those with low incomes (Il). These coffees, with Red, are sometimes drunk (Ds) at work (Ow). In addition, Red is bought always or regularly (Ba, Br), drunk always, often or regularly (Da, Do, Dr). All other coffee brands clutter in the centre of the plot, as do the remaining category-points. Hag (a decafeinated coffee) is seen to occupy a slightly different position. With only seven assessors these observations are extremely tentative, especially when one recalls that most assessors used only about two thirds of the permissible category-levels.



Assessor 1
Il Ow
Bs
Ov
Ds
Ba
Mm
Md Br Da
Dr
Bn
Oh
Dn
Im
Mb
Assessor 2
Bs
Ds
Ml
Md
Br
Dr
Me
Oh
Im
Il
Or Bn
Mm Dn
Od
Ih
Assessor 3
Ov
Mb
Il
Im
Dn
Bn
Bs
Ds
Ih
Br
Dr Mm Oh
Od
Me
Assessor 4
Bn
Ds
Mm
Ow
Il
Im
Oh
Br
Dn
Op
Ih
Me
Md Bs
Mb
Assessor 5
Ov
Ma
Od
Mm
Ih
Ow
Ba
Da
Bn Dn
Ds
Bs
Im
Or
Md
Me
Oh
Assessor 6
Il
Ds
Me
Ov
Da
Bs
Oh
Ba
Md
Mm
Op Ml
Or
Bn
Mb
Im
Ih
Dn
Dimension 2

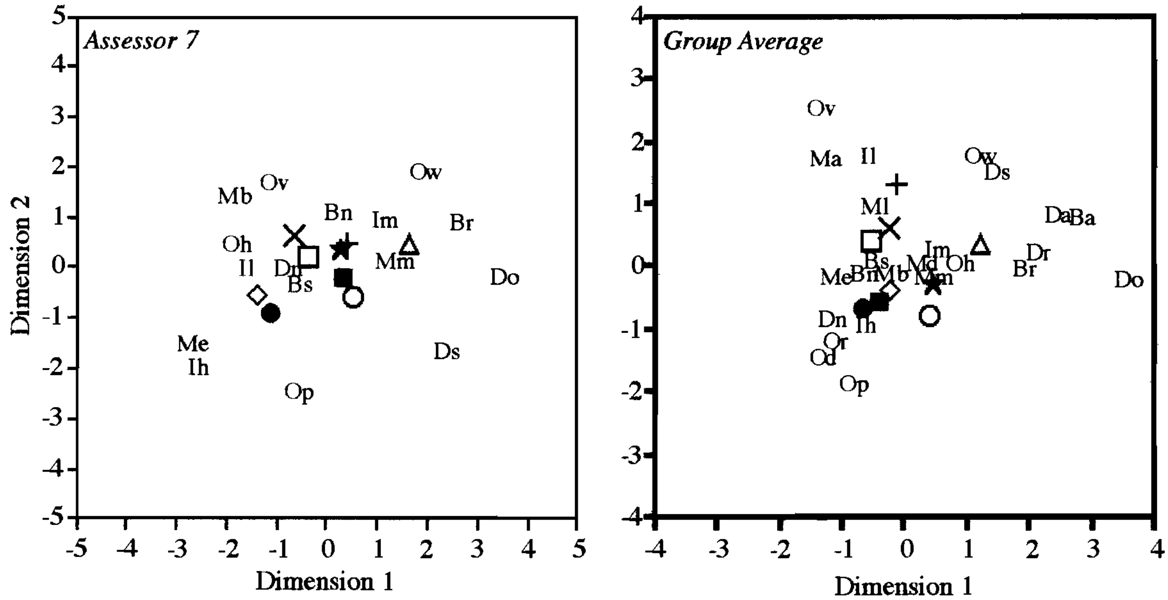



Figure 2
The GPA of each assessor shown relative to the principal axes of the group-average - categorical variables only. Also shown is the group-average itself. The symbols for the coffees are defined in Table 1.

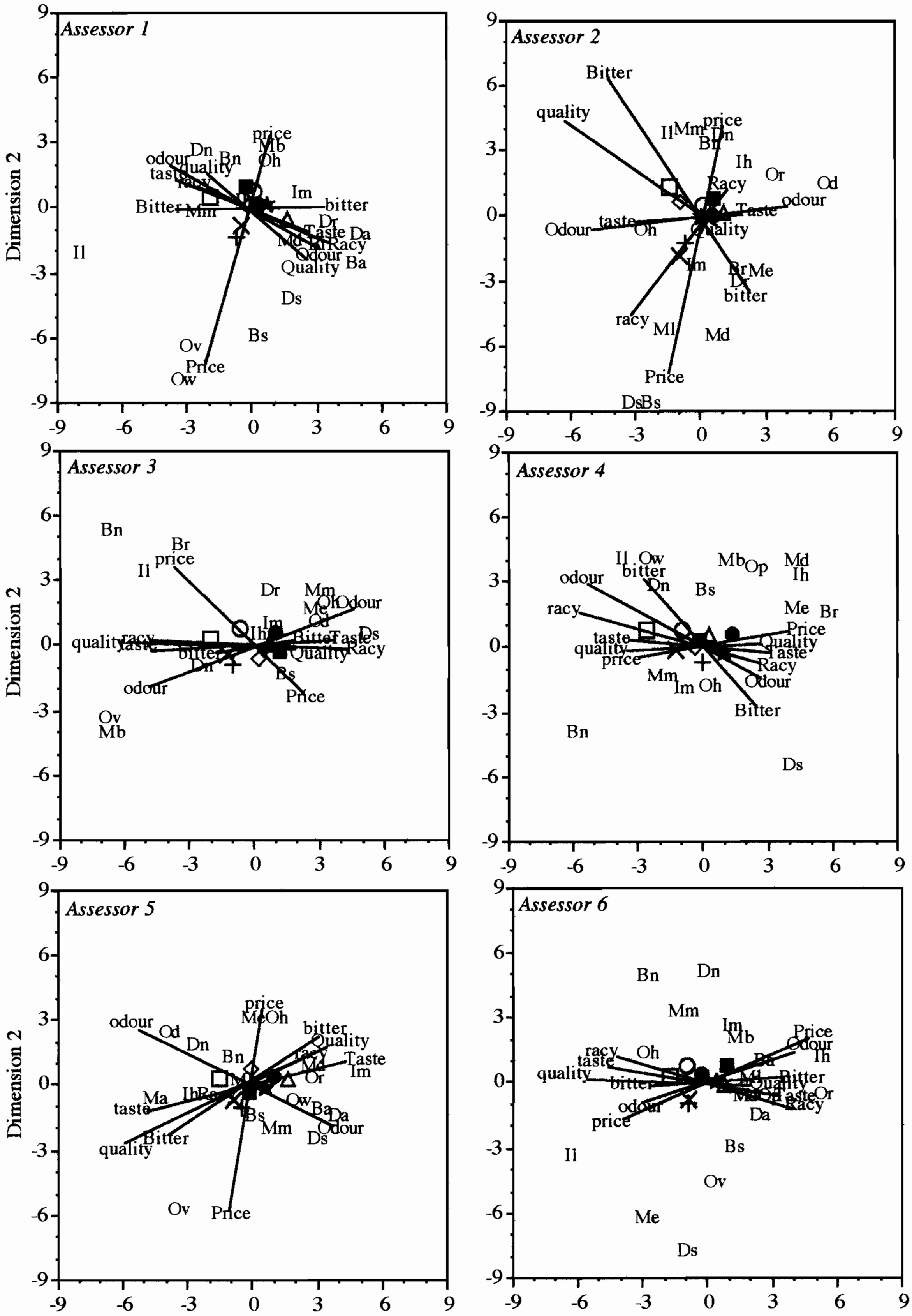
### *4.2. Quantitative and Categorical Variables Combined*

A similar analysis was done using both quantitative and categorical variables. The percentage explained variance for the first two dimensions has been included in Table 3(b) and the associated analysis of variance is in Table 4(b). It shows a similar level of approximation to that given by the categorical variables alone. Figure 3 is the counterpart of Figure 2 but for a GPA based on all variables. Its main difference lies in the inclusion of quantitative variables which induce linear biplot axes. Because the quantitative variable axes extend beyond the positions for the coffee brands the brands unfortunately appear superimposed in the centre of the plot; ideally the plots should be enlarged to put Figure 3 on the same scale as that of Figure 2. Apart from this artifact, the positions of the brands in the two figures compare well.

In Figure 3 the quantitative biplot axes are labelled at the high-score end by the corresponding variable with an initial capital letter, and in lower case at the low-score end. These correspond to the assessors' maximal and minimal scores, respectively. Note that the linear biplot axes are not drawn in the Group Average plot. This could have been done in several ways, averaging scale points corresponding to the same raw score, or averaging the scale points that correspond with the minimum or maximum scale points used. The axes could also have been included in the GPA matching process by averaging over assessors the unit-points on their corresponding biplot-axes, but we do not show this in the figures.

When we examine the linear biplot axes for assessor 3 we see that the quantitative variables quality, racy, taste and bitter all point in the same direction. These attributes are clearly correlated for this assessor. Odour seems to be a little different and it seems as if price is only partly correlated with these attributes. The right-hand side of the plot is characterized by high scores on the quantitative attributes. For assessor 5 we see that coffee, judged to be of high price, is drunk mostly during vacations (Ov). These characterisations apply to the instant coffees Moccona and Nescafe. For this person, high quality coffees score low on bitter and racy, and high on taste.

The most important category points can be seen in the Group Average plot. In general they seem to be the same as in the group average of Figure 2.


Assessor 1
Assessor 2
Assessor 3
Assessor 4
Assessor 5
Assessor 6
Dimension 2
9
6
3
0
-3
-6
-9
price
Price
quality
Quality
odour
Odour
taste
Taste
bitter
Bitter
racy
Racy
Bn
Dn
Mb
Oh
Im
Mm
Dr
Da
Ba
Md
Ds
Bs
Ov
Ow
Il
Ih
Or
Od
Br
Me
Ml
Mm
Op
Ma

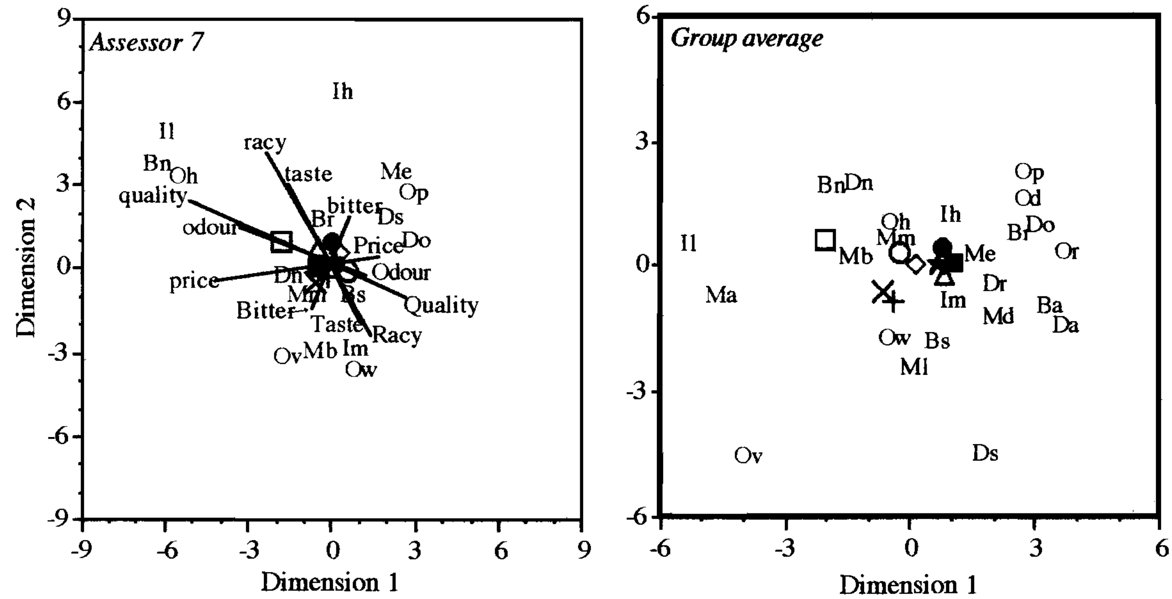
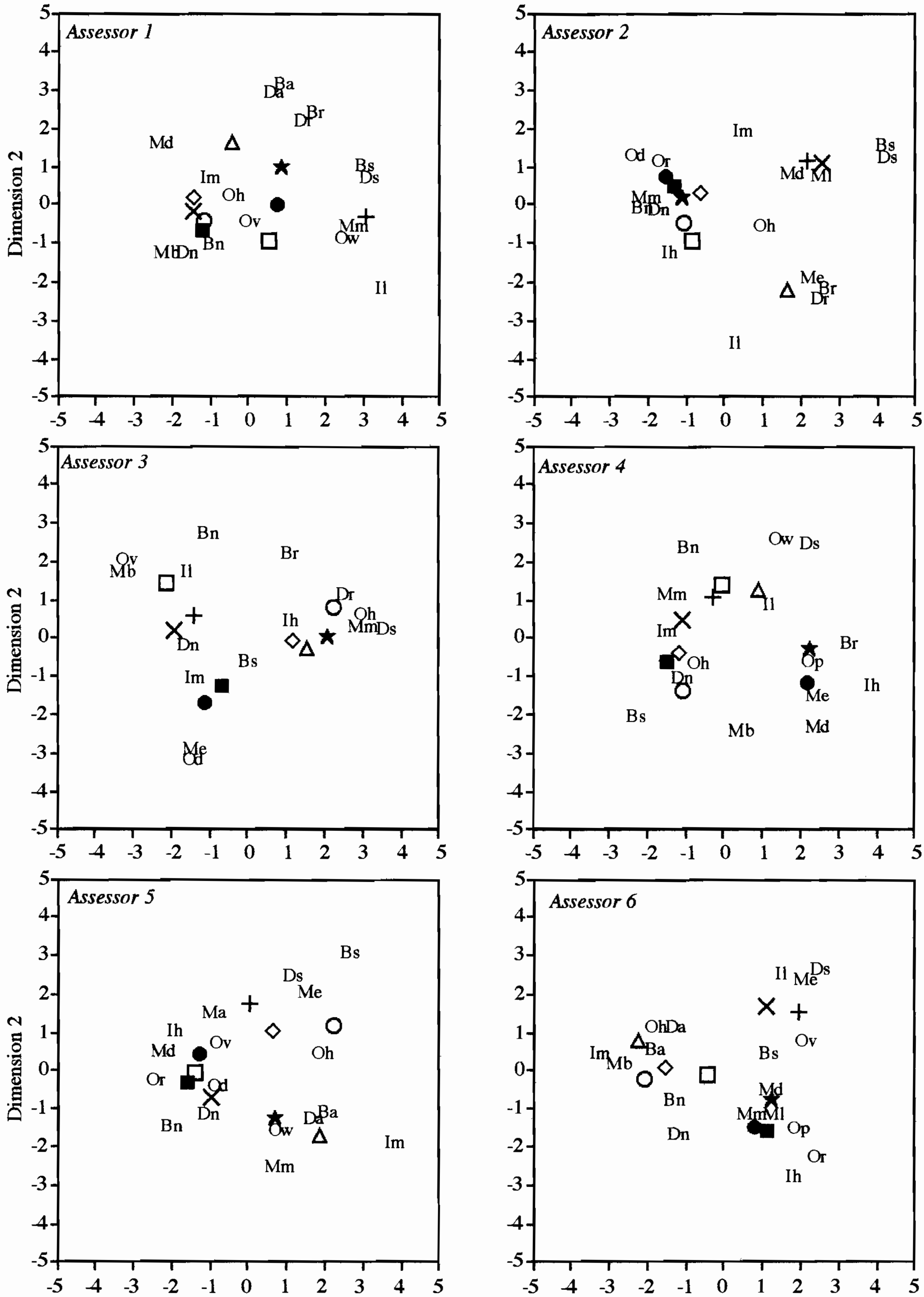



Figure 3
The GPA of each assessor shown relative to the principal axes of the group-average - categorical variables and quantitative variables. Also shown is the group-average itself. The symbols for the coffees are defined in Table 1.

### *4.3. Joint GPA on Brands and Category Levels*

Returning to the analysis of categorical variables given in Section 4.1, we have points for nine brands and a total of 25 category-levels. These are common to all assessors, except that some levels are missing for some assessors. Assuming for the present that all 34 points are available for all assessors, it is clear that a GPA could be done that simultaneously oriented to best fit the information on brands and on category-levels. This would differ from the analysis of Section 4.1 which optimises the fit for the brands, leaving the category-levels to fit in as best they can. The proposed form of representation must give a poorer fit for the brands but it gives a better fit for the category-levels, so might be regarded as a better compromise to exhibiting both types of information. Figure 4 shows the combined analysis. In doing this analysis, some responses to questions about some categorical variables were missing for some assessors. This complicates the computational processes for matching configurations but Commandeur (1991) has discussed the modifications required and which were used in our analysis.



Assessor 1
Assessor 2
Assessor 3
Assessor 4
Assessor 5
Assessor 6
Dimension 2

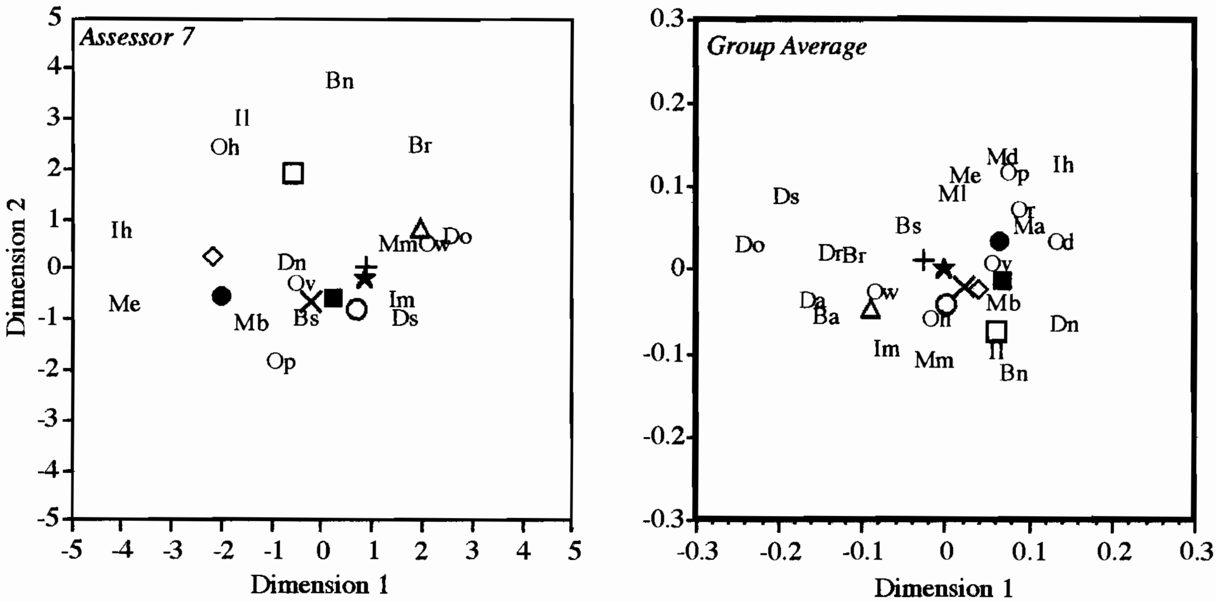



Figure 4
The GPA of each assessor shown relative to the principal axes of the group-average - categorical variables only. Also shown is the group-average itself. In this analysis optimal fit has been obtained using the points representing the brands and the points representing the category-levels. The symbols for the coffees are defined in Table 1.

By including the category-points in the matching process, it is clear that a better overall spread of both the brands and category-points has been achieved. Comparing the assessors' configurations with the ones in figure 2, apart from orientation, reveals similar configurations for both category- and coffee-positions. Table 4c gives the analysis of variance and shows that the exhibited two dimensions give a poorer representation of the coffees. This is the price that has to be paid to accommodate better the information on the category-levels. Nevertheless, on a per dimension basis, the two dimensions are still accounting for much more than the unexhibited dimensions.

Of course, the GPA could also be done solely on the category-level points, thus giving an optimal representation for the variables and leaving the brands to fit as best they can. We could also have done the combined analysis as described in Section 4.2 and include the linear biplots for the quantitative variables. In the latter form of analysis, the linear biplots in the group-average configuration are obtained by averaging like scale points on the axes for the individual assessors. Although both these analyses might sometimes be useful, we did neither but that the possibilities exist exemplifies the flexibility of the methodology.

## 5. Conclusion

The main thing that we have done here is to demonstrate the feasibility of this kind of analysis, especially combining information on quantitative and categorical variables. The few assessors that we have used to demonstrate the methodology would always have been inadequate for a serious investigation into coffee images but the apparent heterogeneity of the responses compounds the difficulty in arriving at any firm conclusions. However, the joint analysis described in Section 4.3 has some attractive features.

In §2 it was pointed out that there might be interest in seeing whether there was evidence that the assessors fell into two or more groups. With only seven assessors, it seemed futile to try to answer this question and, in any case, in a GPA a heterogeneous group-average would tend to obscure such differences, if they existed. It would seem better to proceed by accumulating all the 21 pairwise Procrustes residual sums-of-squares statistics into a $7 \times 7$ symmetric distance-matrix and displaying the seven assessors by some form of multidimensional scaling (Gower, 1971). Then it can readily be seen if the assessors group or not.

### Acknowledgements

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