NONLINEAR ANALYSIS OF MULTIPLICATIVE RULES
IN EXPECTANCY-VALUE MODELS

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Abstract

Expectancy-value models usually prescribe a rule by which scores on variables are added and/or multiplied. However since variables incorporated in an expectancy-value model essentially contain only ordinal information the result of a multiplication depends on more or less arbitrary a priori quantifications of the categories of the variables. This paper proposes a method to circumvent the problems connected to an a priori quantification of ordinal variables that are supposed to interact multiplicatively with each other (in the sense of analysis of variance). It uses a bivariate ordinal measurement level to incorporate a priori theoretical assumptions, implied by expectancy-value models, in nonlinear multivariate analysis. An application is shown in an analysis of the Attitude-behavior model of Fishbein and Ajzen (1975). In the discussion some criticism is ventured with regard to the method and the application shown for illustrative purposes. Some directions for future research are suggested.
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Introduction

Expectancy-value models are wide-spread in social psychology and consumer research. Examples of expectancy-value models are for instance the Attitude-behavior model of Fishbein and Ajzen (1975), the self-explicated utility model of Huber and colleagues (Huber, Sahney, & Ford, 1969; Huber, 1974) and several models for measuring work motivation and performance (Graen, 1969; Vroom, 1964). Expectancy-value models specify a rule of combination between two (or more) predictor variables in the prediction of a criterion variable. Besides a (weighted) summation of predictor variables, as in multiple regression models, pairs or sets of predictor variables are multiplied in order to predict the criterion variable. For instance, in the Attitude-behavior model of Fishbein and Ajzen (1975) it is assumed that the attitude towards an activity (or another object) is influenced by a set of beliefs \( B_i, i = 1..k \), which are the perceived probabilities of certain consequences, about the activity, weighted by the evaluations \( E_i, i = 1..k \) of these consequences. Here \( k \) is the number of salient beliefs. In particular the sum of products of beliefs and evaluations, \( \sum_{i=1}^{k} B_i \times E_i \), is taken in order to predict the attitude \( A \). Fishbein and Ajzen state that this model can be used as long as it has predictive validity, which point of view is of course very pragmatic. The Pearson correlation between the directly measured attitude and \( \sum_{i=1}^{k} B_i \times E_i \), is often taken as a measure of the predictive validity of the model (e.g., Midden, Daamen & Verplanken, 1983). In other instances (e.g., Bagozzi, 1981), a weighted sum of the \( B_i \times E_i \)'s is taken to determine the predictive validity. Several values of predictive validity can be found in literature. Midden et al. (1983) found a Pearson correlation of 0.42 between a directly measured attitude and the sum of products of 23 beliefs about and evaluations of consequences of a wide-scale use of wind to generate electricity. They found a Pearson correlation of 0.63 between the attitude and the sum of products of 34 beliefs about and
evaluations of consequences of a wide-scale use of uranium to generate electricity. Schaeffer, Swaton and Niehaus (1981) found a Pearson correlation of 0.83 between a directly measured attitude and the sum of $B_i \times E_i$'s. Falbo and Becker (1982) wondered, whether the model can be simplified without loosing to much predictive validity. Especially with regard to social relevant topics, there will be consensus about the evaluation of certain consequences and the predictive validity of $\sum_{i=1}^{k} B_i$ will be about equal to the predictive validity of $\sum_{i=1}^{k} B_i \times E_i$. Three criteria seem to be important in order to determine whether the model can be reduced (Midden, 1986): - the variance of each component (that is the belief and the evaluation), - the covariance of the components, and -the covariance with the relevant criterion. An empirical comparison of the original model and the simplified model can be made by computation of their respective predictive validities.

In short, there seems to be much interest in determining a measure of association between a directly measured attitude and the products of beliefs and evaluations. However in practice, multiplication of, empirically obtained, values of beliefs and evaluations is doubtful because it assumes that both beliefs and evaluations are measured on scales that are proportional to the "true subjective values", which is normally not a valid assumption (e.g., Lynch, 1985; Schmidt, 1973). Thus it is necessary to have ratio scaled beliefs and evaluations, which requires the existence of a true rational zero point. Schmidt (1973) shows that the correlation between the product of two, interval scaled, predictor variables and a criterion variable may change drastically if the location of the zero point is changed in one or both predictor variables. Thus it will be difficult to assess the predictive validity of expectancy-value models. In fact, the reduction of the model of Fishbein and Ajzen by Falbo and Becker (1982) partly stems from the desire to circumvent the measurement problem. Cohen (1968, 1978), Cohen and Cohen (1975) and Allison (1977) propose hierarchical multiple regression models to asses the predictive validity in case of interval scaled predictor variables in multiplicative models. Arnold and Evans (1979) and Orth (1987) make some further comments on this topic. However the measurement problem
becomes worse in case of ordinal scaled predictor variables. Here we will give some examples which show that even the rankorder among values, obtained by multiplication of two variables, may change as a result of monotonic transformations of the variables.

**Multiplication and the Effect of Monotonic Transformations**

Suppose we have two predictor variables \( x_1 \) and \( x_2 \) each classifying objects into five ordered categories. In table 1, column I and column II, the categories of the variables are denoted by the letters \( a \) to \( j \) and the rows represent the 25 possible combinations of categories from \( x_1 \) and \( x_2 \). Further suppose that the scales of both variables are conceived of as unipolar. In practice their categories will often be quantified as taking on integer values from 1 to 5. When we multiply \( x_1 \) with \( x_2 \) we obtain 14 different values, namely 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 20 and 25. We can extract a rankorder between combinations of categories from \( x_1 \) and \( x_2 \), according to the values obtained by multiplication of the quantifications. Some of the combinations will have the same value and therefore the rankorder between these categories is not uniquely determined. In column III of table 1 the possible rankorder values for each combination of categories, as a result of the multiplication, are given.

[Insert table 1 about here]

Transformations of the quantifications of \( x_2 \) yield different results of a multiplication of \( x_1 \) and \( x_2 \). When we add 1 as a constant to \( x_2 \), which can be understood as moving our fivepoint scale away from an assumed zero point by a linear transformation, and extract a rankorder, according to the values obtained by multiplication of the (transformed) quantifications, six combinations of categories from \( x_1 \) and \( x_2 \) have a different rankorder value, see column IV of table 1, or different possible rankorder values, compared with the previous rankorder. Taking the logarithm of \( x_2 \) and adding 1 as a constant, which can be understood as the relative stretching and shrinking of intervals of a scale, yields sixteen
different possible rankorder values, see column V of table 1, for combinations of categories from $x_1$ and $x_2$, compared with the original rankorder.

As a second example suppose that the scale of $x_1$ is conceived of as unipolar while the scale $x_2$ is conceived of as bipolar. In practice the categories of $x_1$ will often get quantifications 1, 2, 3, 4 and 5 while the categories of $x_2$ get quantifications -2, -1, 0, 1 and 2. Possible rankorder values for the combinations of categories from $x_1$ and $x_2$, as a result of multiplication, are given in column VI of table 1. If we change the zero point, and the quantifications for the other categories of $x_2$ accordingly, and give the quantifications -1, 0, 1, 2 and 3 to the successive categories we obtain nine different possible rankorder values, see column VII of table 1, as a result of multiplication. Finally, when we assign the quantifications -5, -4, 1, 4 and 5, thus assuming the middlemost category as a little bit positive instead of exactly zero and the differences between on one hand category $f$ and $g$ and on the other hand category $i$ and $j$ to be relatively small, then twelve combinations obtain different rankorder values as a result of multiplication.

Therefore, since variables used in expectancy-value models can at most be supposed to be measured on a interval scale and more likely on a rankorder scale, it should be clear that variables resulting from multiplication as specified in expectancy-value models can not be uniquely determined. This is true for both numerical and ordinal information contained in the resultant variables. As an alternative to a priori quantifications of variables, we might consider to use nonlinear multivariate analysis (MVA) to determine the predictive validity of expectancy-value models.

**Some Features of Nonlinear Multivariate Analysis**

In Gifi (1981) and Young (1981) monotonic and nonlinear versions of some of the more familiar linear multivariate techniques are discussed. In monotonic MVA results are invariant under one-to-one monotonic transformations, whereas in nonlinear MVA results are invariant under all one-to-one nonlinear transformations. Most of the monotonic and nonlinear versions discussed are not entirely monotonic or nonlinear but they offer the opportunity to specify *a priori* the type of invariance for each variable separately in terms
of measurement levels. The various types of measurement levels are numerical, corresponding to invariance under linear transformations, ordinal, corresponding to invariance under monotonic transformations, and nominal, corresponding to invariance under all nonlinear transformations.

One of the basic assumption underlying these techniques is that each variable consists of a finite number, $k_j$, of mutually exclusive and exhaustive categories and that each observation belongs to a particular category on each variable (Gifi, 1981).

Solutions are obtained by minimizing loss functions of the least squares type with a so called alternating least squares method, which alternates between two substeps at each iteration. "In the first substep we compute the optimum basis for given values of the transformations", (of the variables, or for the quantifications of the categories),"in the second substep we compute new values for the optimum transformations for the given basis computed in the first substep" (Gifi, 1981, p.56). We can view this as involving a cyclically repeated series of optimal cone projections which can be proven to converge (De Leeuw, Young, & Takane, 1976).

The nonlinear MVA techniques are largely joint bivariate, which means that interaction between variables (in the sense of Analysis of Variance) is not taken into account. Kruskal (1965) shows that, in the analysis of factorial experiments, the need for certain interaction terms may be eliminated by monotonic transformation of the dependent variable. This idea can be transferred to nonlinear MVA, where transformation of variables may eliminate the need for certain interactions. However in some cases and for some reasons it might still be sensible to permit interactions. In particular this will be the case when interactions are assumed to exist in theory. Introducing interaction into the analysis can be done by "creating a combined variable with as many categories as the number of combinations of categories of the initial variables" (Gifi, 1981, p.74). Such a combined variable will be called an interactive variable. In the next paragraph we will elaborate upon interactive variables.
Interactive Variables in Nonlinear Multivariate Analysis

Suppose two variables are available, $x_1$ and $x_2$, with $k_1$ and $k_2$ categories, that are to be combined into an interactive variable, $x_{12}$, with $k_1 \times k_2$ categories. Let $\gamma_{12j}$ be the optimal quantification for the category of $x_{12}$ that represents the combination of the $i$th category of $x_1$ with the $j$th category of $x_2$. That is, the vector $\gamma_{12}$ contains the scaling parameters for the categories of $x_{12}$, such that, in combination with optimal quantified other variables in the model and optimal model parameters, a specific least squares loss function is minimized. Interaction between $x_1$ and $x_2$ is said to exist when the differences between the quantifications $\gamma_{12j}$’s, for $i \in \{1,...,k_1\}$, depend on the value of $j$ and vice versa. In multiple regression analysis, this means that the contribution of $x_1$ and $x_2$ taken together, to the weighted sum of all predictor variables, does no longer need to be additive. In figure 1 examples are shown of different kinds of interaction by plotting the quantifications $\gamma_{12j}$ against $i$, and by connecting points referring to the same value of $j$.

[Insert figure 1 about here]

In figure 1A there is no interaction between $x_1$ and $x_2$ because the differences between quantifications, $\gamma_{12j}$, for successive $i$’s are the same for all values of $j$, and vice versa. The quantifications are monotonic with respect to values of $i$, but not with respect to values of $j$. In figure 1B there is still no interaction but the quantifications are monotonic with respect to both the values of $i$ and $j$. In figure 1C there exists interaction between $x_1$ and $x_2$ because the differences between quantifications, $\gamma_{12j}$, for successive $i$’s depend on the value of $j$. Quantifications are monotonically increasing with both values of $i$ and $j$, for fixed $j$ respectively fixed $i$. Moreover, differences between successive values of $j$ increase as $i$ increases from 1 to 5 and we can describe the categories of variable $x_1$ as ranging on a dimension from extinguishing the effect of $x_2$ to amplifying the effect of $x_2$.

In figure 1D and 1E there exists interaction between $x_1$ and $x_2$. For some values of $i$,

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1 In order to make clear the relationship between $x_1$ and $x_2$, the categories of $x_1$ and $x_2$, on the one hand and the interactive variable, $x_{12}$, and its categories on the other hand, we use double subscripts for interactive variables.
respectively \( j \), quantifications, \( \gamma_{12ij} \), are monotone increasing while for some values quantifications are monotone decreasing with \( j \), respectively \( i \), and for still other values of \( i \), respectively \( j \), quantifications are not monotonically related to \( j \), respectively \( i \).

It should be noted that, in terms of analysis of variance, the quantifications, \( \gamma_{12ij} \), can be seen as the sum of the overall mean, the main effect terms and the first order interaction term. The overall mean, can be found as, \( \alpha = \sum_{i=1}^{k_1} \sum_{j=1}^{k_2} n_{12ij} / \sum_{i=1}^{k_1} \sum_{j=1}^{k_2} n_{12ij} \). Here, \( n_{12ij} \) denotes the number of observations in category \( i \) of \( x_1 \) and category \( j \) of \( x_2 \). Main effects, \( \alpha_{1i} \), of \( x_1 \) can be found by taking, \( \alpha_{1i} = (\sum_{j=1}^{k_2} n_{12ij} \gamma_{12ij} / \sum_{j=1}^{k_2} n_{12ij}) - \alpha \). Main effects, \( \alpha_{2j} \), of \( x_2 \) can be found by taking, \( \alpha_{2j} = (\sum_{i=1}^{k_1} n_{12ij} \gamma_{12ij} / \sum_{i=1}^{k_1} n_{12ij}) - \alpha \). First order interaction terms can be found by subtraction, \( \alpha_{12ij} = \gamma_{12ij} - (\alpha_{1i} + \alpha_{2j}) + \alpha \). In the analysis of variance, we would not have the optimal quantifications, \( \gamma_{12ij} \), in the equations above, but the cell means of the dependent variable. However, the correspondence between the two can easily be made clear, since the cell means of the dependent variable are optimal estimates in the analysis of variance model that minimize a least squares loss function, namely the residual sum of squares.

Nonlinear Multivariate Analysis of the Model of Fishbein and Ajzen Using Interactive Variables

Before coming to the crux of this paper, we first want to show what kind of results are likely to be obtained when interactive variables are treated at a nominal measurement level in the analysis of expectancy-value models. Therefore, we introduce a study by Midden et al (1983) in which they used the Attitude-behavior model of Fishbein and Ajzen (1975).

Midden et al give a report of "a national survey in the Netherlands on the perception of three options to generate electricity: coal, uranium and wind. Especially the perception
and acceptance of the risks of these energy options have been studied. The questionnaire has been constructed according to a modified Fishbein model and was completed by 1112 respondents. Beliefs and evaluations were rated by the respondents on ninepoint rating scales, attitudes on sevenpoint rating scales. The sum of scores on 34 beliefs about a wide-scale use of uranium to generate electricity, was correlated, after recoding beliefs about negative consequences in the right direction, with the attitude towards a wide-scale use of uranium to generate electricity, and a Pearson correlation of 0.58 was found. This correlation was compared with the Pearson correlation between $\sum_{i=1}^{k} B_i \times E_i$ and the attitude to see if a weighting of beliefs with their evaluations produces a better prediction of the attitude. Quantifying beliefs with integer values from 0 to 8 and evaluations from -4 to 4 a Pearson correlation of 0.63 between $\sum_{i=1}^{k} B_i \times E_i$ and the attitude was found. Midden et al. (1983) conclude there is hardly any improvement in predictive validity, when beliefs are weighted with their respective evaluations. As an explanation for this result they argue that there is not enough variation among respondents in the evaluation of a consequence.

Of thirty-four consequences of the wide-scale use of uranium to generate electricity we selected seven consequences of which the evaluation displayed at least some variation. The descriptions of the seven consequences are: The use of uranium to generate electricity

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1- may lead to storage of nuclear waste in salt domes.
2- may lead to the fact that a lot of people are frightened about the risks of it.
3- may lead to the fact that the local or regional government has more right of say about the energy supply.
4- may lead to a large-scale energy supply, where electricity is generated in a few large power stations.
5- may lead to a small-scale energy supply, where energy is generated in a large number of small power stations located near the consumers.
6- may prevent that the prosperity of the Dutch population will diminish.

7- may lead to the use of a practically inexhaustible source of energy.

To simplify our illustration and to obtain well-filled categories for interactive variables, the original ninepoint rating scales have been transformed to fivepoint scales by taking together adjacent categories. The categories of the resultant belief scales are: -nihil / very small, -small / rather small, -moderate, -rather large / large, -very large / this will surely happen. The categories of the resultant evaluation scales are: -extremely negative / very negative, - rather negative / a little bit negative, -nor negative nor positive, -a little bit positive / rather positive, -very positive / extremely positive. Of all respondents we selected those that did not have any missing values on the variables involved. As a result a sample of 686 respondents has been analysed.

Staying, more or less, in line with Midden et al (1983) the categories of the belief scales can be quantified with integer values from 1 to 5, and the categories of the evaluations with integer values from -2 to 2. When the attitude is predicted from the beliefs by means of linear multiple regression we obtain a multiple correlation coefficient of 0.50. When the attitude is predicted from the beliefs about the consequences multiplied by the evaluations of the consequences, we obtain a multiple correlation coefficient of 0.52, which yields a difference in variance accounted for of two percent. Thus the conclusion of Midden et al (1983) can be sustained for the present selection of consequences. However, it is not possible to determine whether or not this is due to an artefact created by the multiplication of ordinal response values.

As an alternative to the multiplication of a priori quantified beliefs and evaluations, we can consider to combine each pair of belief and evaluation into an interactive variable, and to perform nonlinear multiple regression analysis. Of the measurement levels, mentioned in Gifi (1981) and Young (1981), the nominal measurement level is the only one that can possibly be attached to the interactive variables, because it is not possible to define a complete order among the categories of each interactive variable. A nonlinear multiple regression analysis, with the attitude taken ordinal and each combinations of belief and evaluation nominal, results into a multiple correlation coefficient of 0.74. This nonlinear multiple regression analysis can be seen as an extension of the analysis of variance.
paradigm, used by Bettman, Capon and Lutz (1975) to investigate the multiplicative and additive assumptions in Fishbein's multiattribute-attitude model, from the two-attribute case to the seven-attribute case. However, because of the relatively large number of categories of each interactive variable, the categories will be likely to contain only a few number of observations. This will lead to a capitalization on chance, especially when the number of independent variables in the regression equation increases. This, in its turn, will make the interpretation of patterns of interaction with respect to the expectancy-value model very difficult. For example, optimal nominal quantifications of the interactive variable, constructed by the combination of the belief about and the evaluation of the storage of nuclear waste in salt domes, are shown in a plot in figure 2.

[Insert figure 2 about here]

Assumptions about Monotonic Relations in the Model of Fishbein and Ajzen

In order to reduce the capitalization on chance in nonlinear analysis of the model of Fishbein and Ajzen, some monotonic relations, which do not rely upon a priori quantifications, about the contribution of a belief and the corresponding evaluation in the prediction of an attitude can be derived. However, they do rely upon the assumption that we know the sign of the "true subjective value" of each category of the belief scale and the evaluation scale. They also rely upon the assumption that the multiplicative rule holds for the "true subjective values" in as far as this predicts a cross-over interaction. There is much consensus about the sign of evaluation categories in contrast to the sign of belief categories (e.g., Bettman et al., 1975; Fishbein & Ajzen, 1975). In the current derivation we follow Bettman et al. (1975) and Midden et al. (1983) and assume the belief scale to be unipolar. The monotonic relations derived, are:

I- among people having the same belief, those, who evaluate the consequence more positively, will have a more positive attitude, unless the perceived probability is nil
or zero. In that case there will be no relation between evaluation and attitude, other things being equal,

II- among people, evaluating a consequence as being positive, the ones perceiving the probability as being higher will have a more positive attitude, other things being equal,

III- among people, evaluating a consequence as being negative, the ones perceiving the probability as being higher will have a more negative attitude, other things being equal,

IV- among people, evaluating a consequence as being neutral, there will be no relation between the belief and the attitude, other things being equal.

On basis of the assumptions from which these monotonic relations were derived, we can make a meaningful classification of the categories of beliefs and evaluations into order-reversing, order-ignorant and order-preserving categories. This classification is central to the definition of a bivariate ordinal measurement level as we will see in the next paragraph.

**Interaction under Bivariate Monotonicity Restrictions**

The idea behind a bivariate ordinal measurement level is to permit only those patterns of interaction in the quantifications, that are in line with prior knowledge, e.g. with a theory about the relations in a model. So, we reduce the number of possible patterns of interaction by imposing monotonicity restrictions between quantifications, $\gamma_{12ij}$, at fixed $i$, respectively fixed $j$. For example, as we require quantifications, $\gamma_{12ij}$, to be monotone increasing with $j$, respectively $i$, for fixed $i$'s, respectively $j$'s, then the quantifications plotted in figures 1B and 1C are possible quantifications, while quantifications as plotted in figures 1A, 1D and 1E are not permitted. As stated before, the bivariate monotonicity restrictions for the quantifications $\gamma_{12ij}$ of the categories of $x_{12}$ depend on a conceptual classification of the categories of the two initial ordinal variables. We distinguish three classes for categories of the initial variables, each of which defines specific restrictions on the quantifications of categories of the interactive variable:
1- the \(i^{th}\) category of a variable is \textit{order-reversing} if the order of the quantifications \(\gamma_{12ij}\), with \(j \in \{1,...,k_2\}\), on \(j\) is monotone decreasing or \textit{antitonic}, (Barlow, Bartholomew, Bremner, & Brunk, 1972).

2- the \(i^{th}\) category of a variable is \textit{order-ignorant} if there are no monotonicity restrictions on the quantifications \(\gamma_{12ij}\), with \(j \in \{1,...,k_2\}\).

3- the \(i^{th}\) category of a variable is \textit{order-preserving} if the order of the quantifications \(\gamma_{12ij}\), with \(j \in \{1,...,k_2\}\), on \(j\) is monotone increasing or \textit{isotonic}, (Barlow \textit{et al}, 1972).

Note that the definition of these classes not only presumes monotonicity but also an a priori direction in which the quantifications have to increase. A variable may consist of any number of order-reversing and/or order-ignorant and/or order-preserving categories. Further we assume that the categories of the initial variables are ordered from order-reversing categories to order-ignorant categories to order-preserving categories. Moreover we might call the set of order-ignorant categories of a variable its \textit{turnover-point} since these categories lie in between order-reversing and order-preserving categories.

**Examples of Bivariate Monotonicity Restrictions**

Suppose \(x_1\) and \(x_2\) consist of only order-preserving categories, and both \(k_1\) and \(k_2\) are equal to 5. The bivariate monotonicity restrictions between the \(\gamma_{12ij}\)'s in this case are depicted in table 2.

[Insert table 2 about here]

Suppose \(x_1\) consists of one order-ignorant category and four order-preserving categories and \(x_2\) consists of two order-reversing categories, one order-ignorant category and two order-preserving categories. The bivariate monotonicity restrictions between the \(\gamma_{12ij}\)'s in this case are depicted in table 3.
Some Computational Aspects

The monotonicity restrictions implied by a bivariate ordinal measurement level define a closed convex cone. As a consequence, the overall alternating least squares algorithm still consists of successive steps of optimal cone projections, which implies that the convergence of the algorithm, that minimizes the loss function is guaranteed. To minimize the least squares loss function, we can compute optimal unrestricted quantifications, $\gamma^*_{12iy}$, in each iteration, given the current values of all other parameters in the model, and proceed by computing the least squares projection of the $\gamma^*_{12iy}$'s upon the cone defined by the bivariate monotonicity restrictions.

Bivariate monotonicity restrictions may be considered as defining a special case of the more general class of all possible partial orderings (cf., Van Eeden, 1956). On the other hand the isotonic regression in two independent variables can be considered as a special case of our bivariate monotonic regression problem, namely as both initial variables consist of only order-preserving categories (cf. table 1). Algorithms that yield the exact solution to the isotonic regression problem for two independent variables are found in Gebhardt (1970), Dijkstra (1981) and Barlow et al. (1972). According to Dijkstra and Robertson (1982) these algorithms "all involve search techniques which can require a significant amount of checking and readjustment. Thus computer programs which implement these algorithms require intricate branching logic and are complicated to program". In Van der Lans (1987) two algorithms, based on iterative procedures, are given, which are relatively easy to program and give the least squares estimates of the projection of the $\gamma^*_{12iy}$'s upon the cone defined by the bivariate monotonicity restrictions. One algorithm is more or less similar to an algorithm used by Heiser (1985) to solve his smooth monotonic regression problem. The other algorithm stems from Dijkstra and Robertson (1982) who consider the isotonic regression problem in two independent
variables. In Van der Lans (1987) an outline of these algorithms is given and a comparison, both in precision and computational speed, is made which seems to favour the algorithm based on Dijkstra and Robertson, although the question has not yet been settled.

By definition there are no monotonicity restrictions between the quantifications $\gamma_{12i}$ of an interactive variable, for fixed $i$ and $j \in \{1, 2\}$, when the $i^{th}$ category of the first initial variable is order-ignorant. Thus if one or both of the initial variables contain one or more order-ignorant categories some quantifications are less restricted, which might for example result in extreme values of these quantifications. One possibility to avoid less restricted quantifications is to force them to be equal to each other, or, in other words, to tie them together. In table 4 the restrictions are shown when the less restricted quantifications of table 3 are tied together.

[Insert table 4 about here]

Minor adjustments to the algorithms in order to handle ties are given in Van der Lans (1987).

Nonlinear Multivariate Analysis of the Model of Fishbein and Ajzen Using Bivariate Monotonicity Restrictions with Interactive Variables

To illustrate the use of a bivariate ordinal measurement level in expectancy-values we use the same data from Midden et al (1983), as used earlier in this paper. We also use the same recoding of categories. To specify the bivariate ordinal measurement level all categories of the belief scale are classified as order-preserving while the first two categories of the evaluation scale are classified as order-reversing, the third category as order-ignorant and the last two categories as order-preserving. The resultant bivariate monotonicity restrictions are given in table 5.
By means of nonlinear multiple regression analysis\(^2\) the prediction of the attitude towards the wide-scale use of uranium was analysed. The prediction of the attitude by the beliefs mentioned yields a multiple correlation coefficient of 0.54, when the attitude and the beliefs are treated ordinal. Combining for each consequence the belief and the evaluation and treating the resultant interactive variables as bivariate ordinal gives a multiple correlation coefficient of 0.63. As can be seen, by combining beliefs and evaluations per consequence the percentage of variance accounted for in the attitude increases from 29.2 percent to 39.7 percent. Optimal quantifications for categories of the attitude in the two nonlinear multiple regression analyses are shown in figure 3, together with optimal quantifications obtained in the analysis in which the interactive variables are treated nominal.

As can be seen optimal quantifications for categories of the attitude are very much the same, and almost linear, in the three analyses. In figure 4a to 4c optimal quantifications, obtained in the regression analysis in which the seven beliefs are the independent variables, of belief categories of three consequences are plotted together with optimal quantifications of categories of the interactive variables, obtained in the regression analysis in which seven combinations of beliefs and evaluations are the independent variables. Standardized regression weights (\(\beta\)) and Pearson correlation coefficients (\(r\)) between the optimally quantified attitude and optimally quantified independent variables are also given. The beliefs about these three consequences correlate the highest of all seven beliefs with the attitude. Optimal quantifications, standardized regression weights and Pearson correlation coefficients for the other consequences are given in Van der Lans (1987). We note that optimal quantifications for the interactive variables are indeed

\(^2\) To perform the nonlinear multiple regression analysis a prototype of a computer program based on the MORALS algorithm by Young, De Leeuw and Takane (1976) has been used, written in Fortran77. The bivariate monotonic regression routine similar to the procedure followed by Heiser (1985) has been implemented in this program.
bivariate ordinal, whilst categories corresponding to the combination of belief categories with the order-ignorant evaluation category, nor negative nor positive, are not tied together. Considering optimal quantifications with respect to the storage of nuclear waste in salt domes it can be seen that, because most respondents evaluate this consequence negatively, optimal quantifications for the uncombined belief resemble the optimal quantifications of the categories of the interactive variable for those respondents that have a negative evaluation, though quantifications for the uncombined belief are somewhat higher. As well as for the uncombined belief as for the categories of the interactive variable for those respondents that have a negative evaluation, the stronger the belief the more negative the attitude. This might also be argued for those respondents that evaluate this consequence nor negatively nor positively, however for them the relationship is much weaker and therefore less clear.

[Insert figure 4a to 4c about here]

Among respondents that evaluate the storage of nuclear waste in salt domes positively those having the stronger belief do not seem to have a more positive attitude than those having a less strong belief, which in some sense contradicts our second assumption about the interaction between beliefs and corresponding evaluations in the prediction of an attitude. Perhaps this is due to a ceiling effect in that those people holding a positive evaluation and a weak belief have a very positive attitude and therefore it is impossible for other groups to have a more positive attitude.

Observing figure 4b perhaps the most striking is the fact that the correlation with the optimally quantified attitude is much higher for the optimally quantified uncombined belief than for the combined belief and evaluation. The reason for this might be that, again in contradiction with our second assumption about the interaction between beliefs and corresponding evaluations in the prediction of an attitude, among respondents evaluating the fact that a lot of people will be frightened about the risks as being positive those with a stronger belief have a more negative attitude. A possible explanation for this is that respondents having a more negative attitude reason that fears about risks will enhance vigilance and resistance with respect to the use of uranium (Midden et al., 1983, p. 186).
Considering this it is important to mark that the evaluation is not a correct evaluation in the Fishbeinian sense since reference is made to the object of attitude. The reason for this is that the researchers in some cases doubt the meaningfulness of aspecific evaluations (personnel communication) and therefore formulated the evaluation on a more specific level, see also Ahtola (1975). However in Midden (1986, p. 208) it is noted, based on a linear multiple regression analysis, that the $B_i x E_i$-score for the fact that a lot of people will be frightened about the risks doesn't contribute much to the prediction of the attitude without mentioning any explanation. We argue that, in contrast to the linear multiple regression analysis performed by Midden (1986), in our nonlinear multiple regression analysis with a bivariate ordinal measurement level it is possible to gain insight in the reason for the small contribution to the prediction of the attitude.

Figure 4c shows that the Pearson correlation of the optimally quantified attitude with the optimally quantified combination of belief and evaluation is higher than with the optimally quantified uncombined belief. It might be argued that their exists an amplifying effect of the evaluation with respect to the relation between belief and attitude. For instance, among respondents evaluating the attribute very/extremely positive the relation between belief and attitude is stronger in comparison with respondents evaluating it a little bit/rather positive. Although a relation between belief and attitude among those respondents evaluating the attribute nor negative nor positive is not to be expected (compare for the assumptions about the interaction between a belief and the corresponding evaluation) we observe that among these respondents those holding a belief ranging from rather large to "this will surely happen" have a relative positive attitude compared to those having a belief ranging from nihil to moderate. Therefore it might prove worthy to reconsider the existence, place on the scale and meaning of a zero point. That is to say, is it possible to determine an evaluation category at which there is no relation between belief and attitude?

When we require categories obtained by combining successive belief categories with a neutral, that is, neither negative nor positive, evaluation to be tied, thereby prohibiting a relation between belief and attitude among respondents holding a neutral evaluation, there's only a very small decrease in the multiple correlation coefficient. The multiple correlation coefficient is 0.62. It can be concluded that the predictive value of these seven combinations of beliefs and evaluations together is hardly diminished when the category
"nor positive nor negative" is considered as the zero point of the evaluation scale. However it will still be possible that the predictive value of individual combinations of beliefs and evaluations is diminished. To study this Pearson correlations between a combination of a belief and an evaluation and the attitude in the two cases should be compared.

Discussion

In the previous illustration we tried to show that a bivariate ordinal measurement level used in nonlinear MVA can be of help in studying interaction between two ordinal variables in relation with a third variable (or more generally in relation with other variables). Incorporating theoretical considerations about the pattern of interaction into the analysis provides us with more stable and interpretable results (Srinivasan, Jain, & Malhotra, 1983). Some flexibility is provided in defining order-ignorant categories which either do not impose any restrictions between quantifications of categories or require quantifications to be equal. In the last case the order-ignorant category is restricted to be effective zero (cf., Krantz & Tversky, 1971).

We have restricted ourselves to the specification of a bivariate ordinal measurement level for discrete data for which the underlying process, which generated the observations, is assumed to be discrete, De Leeuw et al. (1976). This implies that observations in one category are to remain tied (Kruskal (1964) called this the secondary approach to ties). There seems to be no special problem in generalizing bivariate monotonicity to the case in which the underlying process is assumed to be continuous. In that case tied observations are to be untied (Kruskal called this the primary approach to ties). When smooth regression, as proposed by Heiser (1985) in the field of multidimensional scaling, turns out to be useful in nonlinear multivariate analysis we might replace bivariate monotonicity restrictions by bivariate smoothness restrictions or use them complementary to each other. Since as noted by Kruskal (1965) "monotonicity seldom represents all that we wish to
assume about $f$ [the monotonic transformation function]. Almost always we also believe that $f$ is smooth ".

Besides the present use of a bivariate ordinal measurement level in nonlinear multiple regression analysis, other applications are also possible and perhaps fruitful. For instance, the bivariate ordinal measurement level might be used in conjoint analysis (Green & Srinivasan, 1978) to incorporate a priori knowledge of the ordering of part-worths for different levels of an attribute (Srinivasan, Jain & Malhotra, 1983). With the use of a bivariate ordinal measurement level the sum of the main effects and the first order interaction effect of two attributes can be required to obey the a priori bivariate monotonicity restrictions.

Yet, an other application of a bivariate ordinal measurement level could be in nonlinear principal components analysis. Here, it can be used to investigate whether expectancy-value measures of attitude converge to indicate an one dimensional attitude (Bagozzi, 1981; Verhallen, & Pieters, 1984), without multiplying more or less arbitrary a priori scale values.

Although the current method seems to offer a fertile approach to the analysis of expectancy-value models, it has some less attractive features as well. Especially in the application given as an illustration of the use of a bivariate ordinal measurement level, some issues which certainly need careful consideration were by-passed without mention. We will now address these issues, which concern the definition of order-ignorant categories, the correspondence between multiplicative rules and bivariate monotonicity, different transformations of equivalent variables, the selection of consequences, and the analysis of the model across respondents.

A bivariate ordinal measurement level circumvents the problem involved in multiplying variables which essentially contain only ordinal information. Nonetheless an a priori classification of categories is still necessary. This classification should follow from theoretical considerations (cf. Fishbein & Ajzen, 1975), which can be seen as either an advantage or a disadvantage of the method. With regard to order-ignorant categories it
should be remarked that when no restrictions are imposed between quantifications of categories associated with an order-ignorant category of one of the initial variables, there is typically too much freedom left in the estimation of quantifications. On the other hand when the quantifications of these categories are restricted to be equal there is too little freedom left in the estimation of quantifications. A third alternative would be to require a monotonic (that is either isotonic or antitonic) relation between quantifications of categories associated with an order-ignorant category.

Despite the remark of Krantz and Tversky (1971) that "instances in which sign dependence and generalized cancellation hold", (which is implied by bivariate monotonicity restrictions) "in which the composition rule is different from the multiplicative rule are very rare", there is no guarantee that the optimal quantifications of the categories of a bivariate ordinal variable can be written as the result of multiplication of quantifications of the categories of the initial variables. We would stay much closer to multiplicative rules in expectancy-value models if we computed optimal quantifications of the categories of the initial variables, which, multiplied, yield optimal "multiplicative" quantifications of the interactive variable. Research in this direction is currently being undertaken. This truly multiplicative approach also releases us from possible problems associated with the definition of order-ignorant categories.

It might have occurred to attentive readers of this paper that we use nonlinear multiple regression analysis to analyse expectancy-value models whereas these models usually prescribe an unweighted sum of product terms (cf. Fishbein and Ajzen, 1975). However, these unweighted sums of product terms are computed from a priori quantifications of the initial variables. The a priori quantifications are always chosen such that variables of the same kind obtain the same quantifications. For example, in order to compute the sum of products of beliefs and evaluations in the model of Fishbein and Ajzen, all beliefs about consequences are assigned the same a priori quantifications. This implies that consequences about which beliefs differ considerably will be likely to contribute more to differences in attitude than consequences about which the beliefs show much consensus. The same argument counts with respect to the variation in evaluations. Unfortunately,
since the nonlinear analyses used in this paper allow for different transformations of the predictor variables this property does not hold in case we compute the unweighted sum of the optimal quantifications of bivariate ordinal predictor variables. An unweighted sum of the normalized (variance equal to one) predictor variables gives each predictor variable an equal contribution to differences in attitudes whether there is consensus in beliefs and/or evaluations or not. In the light of expectancy-value models this is not desirable. Given different optimal transformations of predictor variables there is no other sensible way to determine the contributions of the predictor variables relative to each other than to estimate regression weights which can be done in multiple regression analysis. An evident disadvantage of using multiple regression is that it becomes possible for predictor variables which show little spread in evaluation and belief to contribute more (account for more variance in) to differences in attitude than predictor variables which show a lot of spread in evaluation and belief. This will complicate the interpretation of the results. Taking into account the instability of regression weights this is not a problem to be taken lightly. A solution to this problem of different transformations might be to restrict the transformations of the predictor variables to be equal. In that case it makes sense again to take unweighted sums of products. This is analogous to a modification of expectancy-value models proposed by Orth (1987) in case the initial variables are assumed to be interval scaled. Orth computes an optimal linear transformation which is equal across variables of the same kind.

In the application given seven consequences were selected of which the evaluation showed at some variation. For illustrative purposes this choice is not the worst that could have been made. However, the model of Fishbein and Ajzen (1975) states that the attitude is to be determined as the sum of products of salient beliefs and their corresponding evaluations. There is no reason at all that the beliefs about the consequences that we have chosen in our application are salient. It might even be that the salient beliefs in the study by Midden et al. (1983) concern social relevant topics about which there is in general much consensus with regard to the evaluation (Falbo & Becker, 1982). To illustrate the practical value of a bivariate ordinal measurement level studies are needed in which
consequences for which beliefs are salient show some spread in belief as well as in evaluation.

Another problem associated with the application in this paper is that analyses are performed across respondents. Several authors have questioned the appropriateness of this approach and advocate individual level analyses (cf. Wilkie & Pessemer, 1973; Bettman et al, 1975; Mitchell, 1982). Considering their arguments we also advocate individual level analyses. These individual level analyses are possible only if we have more than one object of attitude. Since all data for one analysis should be obtained from one respondent there will typically be few data points relative to the number of parameters to be estimated. This problem becomes worse in nonlinear multiple regression analysis and will lead to capitalization on chance and instability of estimates. Future research to nonlinear MVA of expectancy-value models should take this aspect into account and look for ways in which the number of parameters to be estimated can be reduced.

References


Table 1

Rankorders as results of multiplication of two variables, which take on several different quantifications.

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Figure 1
Examples of possible quantifications of an interactive variable

Figure 1A

Figure 1B

Figure 1C

Figure 1D

Figure 1E
Figure 2
Optimal nominal quantifications for belief about and evaluation of the storage of nuclear waste in salt domes.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>◆</td>
<td>extremely/very negative</td>
</tr>
<tr>
<td>▲</td>
<td>rather/a little bit negative</td>
</tr>
<tr>
<td>▧</td>
<td>nor negative nor positive</td>
</tr>
<tr>
<td>○</td>
<td>a little bit/rather positive</td>
</tr>
<tr>
<td>■</td>
<td>very/extremely positive</td>
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</table>

- belief
  - nihil/very small
  - small/rather small
  - moderate
  - rather large/large
  - very large/this will surely happen

- quantification
  - 6
  - 4
  - 2
  - 0
  - -2
  - -4
Table 2
Example 1 of bivariate monotonicity restrictions.

\[
\begin{array}{cccccc}
\gamma_{11} & \leq & \gamma_{12} & \leq & \gamma_{13} & \leq \gamma_{14} & \leq \gamma_{15} \\
1\Lambda & 1\Lambda & 1\Lambda & 1\Lambda & 1\Lambda \\
\gamma_{21} & \leq & \gamma_{22} & \leq & \gamma_{23} & \leq \gamma_{24} & \leq \gamma_{25} \\
1\Lambda & 1\Lambda & 1\Lambda & 1\Lambda & 1\Lambda \\
\gamma_{31} & \leq & \gamma_{32} & \leq & \gamma_{33} & \leq \gamma_{34} & \leq \gamma_{35} \\
1\Lambda & 1\Lambda & 1\Lambda & 1\Lambda & 1\Lambda \\
\gamma_{41} & \leq & \gamma_{42} & \leq & \gamma_{43} & \leq \gamma_{44} & \leq \gamma_{45} \\
1\Lambda & 1\Lambda & 1\Lambda & 1\Lambda & 1\Lambda \\
\gamma_{51} & \leq & \gamma_{52} & \leq & \gamma_{53} & \leq \gamma_{54} & \leq \gamma_{55} \\
1\Lambda & 1\Lambda & 1\Lambda & 1\Lambda & 1\Lambda \\
\end{array}
\]
Table 3
Example 2 of bivariate monotonicity restrictions.

<table>
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<th>( \gamma_{11} )</th>
<th>( \gamma_{12} )</th>
<th>( \gamma_{13} )</th>
<th>( \gamma_{14} )</th>
<th>( \gamma_{15} )</th>
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<td>IV</td>
<td>IV</td>
<td>IA</td>
<td>IA</td>
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<tr>
<td>IV</td>
<td>IV</td>
<td>IA</td>
<td>IA</td>
<td></td>
</tr>
<tr>
<td>( \gamma_{31} \leq \gamma_{32} \leq \gamma_{33} \leq \gamma_{34} \leq \gamma_{35} )</td>
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<td></td>
</tr>
<tr>
<td>IV</td>
<td>IV</td>
<td>IA</td>
<td>IA</td>
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</tr>
<tr>
<td>( \gamma_{41} \leq \gamma_{42} \leq \gamma_{43} \leq \gamma_{44} \leq \gamma_{45} )</td>
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<td></td>
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<tr>
<td>IV</td>
<td>IV</td>
<td>IA</td>
<td>IA</td>
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<tr>
<td>( \gamma_{51} \leq \gamma_{52} \leq \gamma_{53} \leq \gamma_{54} \leq \gamma_{55} )</td>
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</table>
Table 4
Example of bivariate monotonicity restrictions in combination with ties.

\[ \begin{align*}
\gamma_{11} &= \gamma_{12} = \gamma_{13} = \gamma_{14} = \gamma_{15} \\
IV &\quad IV &\quad II &\quad IA &\quad IA \\
\gamma_{21} \leq \gamma_{22} \leq \gamma_{23} \leq \gamma_{24} \leq \gamma_{25} \\
IV &\quad IV &\quad II &\quad IA &\quad IA \\
\gamma_{31} \leq \gamma_{32} \leq \gamma_{33} \leq \gamma_{34} \leq \gamma_{35} \\
IV &\quad IV &\quad II &\quad IA &\quad IA \\
\gamma_{41} \leq \gamma_{42} \leq \gamma_{43} \leq \gamma_{44} \leq \gamma_{45} \\
IV &\quad IV &\quad II &\quad IA &\quad IA \\
\gamma_{51} \leq \gamma_{52} \leq \gamma_{53} \leq \gamma_{54} \leq \gamma_{55}
\end{align*} \]
Table 5
Bivariate monotonicity restrictions upon the quantifications of an interactive variable, obtained by combining the belief about and the evaluation of a certain consequence.

<table>
<thead>
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<th>belief</th>
<th>evaluation</th>
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</thead>
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</tr>
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<td>nihil/very small</td>
<td>$\gamma$ 11 $\leq$ $\gamma$ 12 $\leq$ $\gamma$ 13 $\leq$ $\gamma$ 14 $\leq$ $\gamma$ 15</td>
</tr>
<tr>
<td>small/rather small</td>
<td>$\gamma$ 21 $\leq$ $\gamma$ 22 $\leq$ $\gamma$ 23 $\leq$ $\gamma$ 24 $\leq$ $\gamma$ 25</td>
</tr>
<tr>
<td>moderate</td>
<td>$\gamma$ 31 $\leq$ $\gamma$ 32 $\leq$ $\gamma$ 33 $\leq$ $\gamma$ 34 $\leq$ $\gamma$ 35</td>
</tr>
<tr>
<td>rather large/large</td>
<td>$\gamma$ 41 $\leq$ $\gamma$ 42 $\leq$ $\gamma$ 43 $\leq$ $\gamma$ 44 $\leq$ $\gamma$ 45</td>
</tr>
<tr>
<td>very large/this will surely happen</td>
<td>$\gamma$ 51 $\leq$ $\gamma$ 52 $\leq$ $\gamma$ 53 $\leq$ $\gamma$ 54 $\leq$ $\gamma$ 55</td>
</tr>
</tbody>
</table>
Figure 3

Optimal ordinal quantifications of the attitude towards a wide-scale use of uranium to generate electricity.

- Combined beliefs and evaluations treated nominal
- Beliefs treated ordinal
- Combined beliefs and evaluations treated bivariate
  ordinal

Quantification

Attitude

very bad  bad  a little  nor  a little  good  very good

3 2 1 0 -1 -2
Legend figure 4a to 4c

- uncombined belief
- combined beliefs and evaluations
- evaluation extremely/very negative
- evaluation rather/a little bit negative
- evaluation nor negative nor positive
- evaluation a little bit/rather positive
- evaluation very/extremely positive

Figure 4a
Optimal quantifications for belief about and evaluation of the storage of nuclear waste in salt domes.

$\beta$ (beliefs only) = 0.23
$\beta$ (combined beliefs and evaluations) = 0.39
$r$ (beliefs only) = 0.35
$r$ (combined beliefs and evaluations) = 0.52
**Figure 4b**

Optimal quantifications for belief about and evaluation of the fact that a lot of people will be frightened about the risks.

\[ \beta \text{ (beliefs only)} = 0.34 \]
\[ \beta \text{ (combined beliefs and evaluations)} = 0.18 \]
\[ r \text{ (beliefs only)} = 0.44 \]
\[ r \text{ (combined beliefs and evaluations)} = 0.35 \]
Figure 4c
Optimal quantifications for belief about and evaluation of a practically inexhaustible source of energy.

\[ \beta \text{ (beliefs only)} = 0.23 \]
\[ \beta \text{ (combined beliefs and evaluations)} = 0.24 \]
\[ r \text{ (beliefs only)} = 0.28 \]
\[ r \text{ (combined beliefs and evaluations)} = 0.38 \]